Ratio and Proportion:

Important facts and formula:

1. Ratio:

The ratio of two quantities 'a' and 'b' in the same units, is the fraction $\frac{a}{b}$ and we write it as a : b

In the ratio a : b, we call a as the first term or antecedent and b, the second term or consequent.

Example:

The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

2. **Proportion:**

The equality of two ratios is called Proportion.

If a : b = c : d, we write a : b :: c : d and we say that a, b, c, d are in proportion. Here 'a' and 'b' are called extremes while b and c are called mean terms.

Product of means = Product of extremes.

Thus $a : b :: c : d \iff b \times c = a \times d$

3\.

Fourth Proportional:

(i) If $a : b = c : d$ then d is called the fourth proportional to a, b, c

(ii) Third Proportional: If $a : b = b : c$ then c is third proportional to a and b

(iii) Mean proportional between 'a' and 'b' is $\sqrt{ab}$

4\.

i) Comparison of ratios:

we say that $(a : b) > (c : d)$

$$\iff \frac{a}{b} > \frac{c}{d}$$

(ii) compounded ratio:

The compounded ratio of the ratios $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$

5)

(i) Dulicate ratio of $(a:b)$ is $(a^2:b^2)$

(ii) Sub-dulicate ratio of $(a:b)$ is $(\sqrt{a}:\sqrt{b})$

(iii) Triplicate ratio of $(a:b)$ is $(a^3:b^3)$

iv) Sub-triplicate ratio of $(a:b)$ is $(a^{1/3}:b^{1/3})$

v) If $\frac{a}{b} = \frac{c}{d}$, the $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

6) Variation:

i) We say that x is directly proportional to y; if $x = ky$ for some constant k we write, $x \propto y$.

(ii) We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$

Problem:

1. If $a:b = 5:9$ and $b:c = 4:7$ find $a:b:c$

Sol:

$a:b = 5:9 \qquad b:c = 4:7$

$b:c = (4 \cdot \frac{9}{4} : 7 \cdot \frac{9}{4}) = 9 : \frac{63}{4}$

$a : b : c = 5 : 9 : \frac{63}{4}$

2. Find

(i) The fourth proportional to 4, 9, 12

(ii) The third proportional to 16 and 36

(iii) The mean proportional between 0.08 and 0.18

Sol:

(i) Let the fourth proportional to 4, 9, 12 be x

$4 : 9 :: 12 : x$

$4 : 9 = 12 : x$

$\frac{4}{9} = \frac{12}{x}$

$4x = 9 \times 12 \; 3$

$x = 9 \times 3$

$x = 27$

(ii) Let the third proportional to 16 and 36 be x

$16 : 36 :: 36 : x$

$\frac{16}{36} = \frac{36}{x}$

$16x = 36 \times 36$

$x = \frac{36 \times 36}{16} = 81$

Third proportional to 16 and 36 is 81.

(iii) Mean Proportional between 0.08 and 0.18

sol:

$= \sqrt{ab} = \sqrt{0.08 \times 0.18}$

$= \sqrt{\frac{8}{100} \times \frac{18}{100}}$

$= \sqrt{\frac{144}{100 \times 100}} = \frac{12}{100} = 0.12$

3. If $x : y = 3 : 4$ find $(4x+5y) : (5x-2y)$

sol:

$\frac{x}{y} = \frac{3}{4}$

$\frac{4x+5y}{5x-2y} = \frac{y(4\frac{x}{y}+5)}{y(5\frac{x}{y}-2)}$

$= \frac{4\frac{x}{y}+5}{5\frac{x}{y}-2}$

$= \frac{4\left(\frac{3}{4}\right)+5}{5\left(\frac{3}{4}\right)-2}$

$$= \frac{8}{\frac{15-8}{4}} = \frac{8\times4}{7} = \frac{32}{7}$$

4) Divide Rs. 672 in the ratio 5 : 3

Ans:

Total = 5+3 = 8

First part $= \frac{5}{8} \times 672 = 5\times84$

$= 420$

Second part $= \frac{3}{8} \times 672 = 3\times84 =$

$= 252.$

5. Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.

Sol:

35 + 28 + 20 = 83

A's share $= \frac{35}{83} \times 1162 = 490$

B's share $= \frac{28}{83} \times 1162 = 28\times14 = 392$

C's share $= \frac{20}{83} \times 1162 = 20\times14 = 280$

6. A bag contains 50P, 25P and 10P coins in the ratio 5 : 9 : 4 amounting to Rs. 206 Find the number of coins.

Sol:

Let the number of 50P, 25P and 10P coins be $5x$, $9x$ and $4x$ respectively.

In one rupee = 100p

100p = 1 Re

50p = $\frac{1}{2}$ Re

25p = $\frac{1}{4}$ Re

10p = $\frac{1}{10}$ Re

$$5x \times \frac{1}{2} + 9x \times \frac{1}{4} + 4x \times \frac{1}{10} = 206.$$

$$\frac{5x}{2} + \frac{9x}{4} + \frac{4x}{10} = 206$$

LCM = 2 × 2 × 5 = 20

2	2,	4,	10
2	1,	2,	5
5	1,	1,	5
	1	1	1

$$\frac{50x}{20} + \frac{45x}{20} + \frac{8x}{20} = 206$$

$$\frac{50x + 45x + 8x}{20} = 206$$

$$103x = 206 \times 20$$

$$103x = 4120$$

$$x = \frac{4120}{103} = 40$$

Number of 50p Coins = 5 × 40 = 200

Number of 25p coins = 9 × 40 = 360

Number of 10p Coins = 4 × 40 = 160

7. A Mixture contains alcohol and water in the ratio 4 : 3. If 5 litres of water is added to the mixture, the ratio becomes 4:5 Find the quantity of alcohol in the given mixture.

Sol:

Let the mixture contains alcohol and water be 4x, 3x respectively

$$\frac{4x}{3x+5} = \frac{4}{5}$$

$$4x(5) = 4(3x+5)$$

$$20x = 12x + 20$$

$$20x - 12x = 20$$

$$8x = 20$$

$$x = \frac{20}{8}$$

$x = \frac{5}{2}$

4

The quantity of alcohol is $4\left(\frac{5}{2}\right)$

$= 10$

PROBLEMS BASED ON PERCENTAGE

PERCENTAGE –GAIN AND LOSSS

NOTES-II

- 9.By selling 33 metres of cloth one gain the selling price of 11 metres. Find the gain percent.
- Sol:
- (S.P of 33m)-(C.P of 33m)=S.P of 11m
- S.P of 33m-S.P of 11m=C.P of 33m
- S.P of 22m =C.P of 33m
- Let C.P of each metre be Rs.1
- Then , C.P of 22m=RS.22
- S.P of 22m=Rs.33
- S.P of 22m =Rs.33
- Gain= $\left[\frac{11}{22}\times 100\right] = 50\%$

- 10) A vendor bought bananas at 6 for Rs.10 and sold them at 4 for Rs.6.Find his gain or loss percent?
- Sol:
- Suppose number of bananas bought= L.C.M of 6 and 4
- = 12

$$C.P = Rs.\left[\frac{10}{6}\times 12\right] = RS.20$$

$$S.P = RS.\frac{6}{4}\times 12 = RS.18$$

$$loss = \left[\frac{20-18}{20}\times 100\right]\%$$

$$= \left[\frac{2}{20}\times 100\right] = 10\%$$

3	6,4
2	2,4
2	1,2
	1,1

- 11) A man bought toffees at 3 for a rupee. How many for a rupee must be sell to gain 50%?
- Sol:
- C.P of 3 toffees = Re.1
- S.P of 3 toffees =150% of Re.1 = $\frac{3}{2}$
- For Rs $\frac{3}{2}$ toffees sold =3.
- For Re.1 , toffees sold= $\left[3 \times \frac{2}{3}\right] = 2$

- 12) A grocer purchased 80 kg of sugar at 13.50 per kg and mixed it with 120 kg sugar at Rs.16 per kg .At what rate should be sell the mixture to gain 16%?
- Sol:
- C.P of 200 kg of mixture =[Rs.(80×13.50)+120×16]
- =Rs.3000
- S.P=116% of RS.3000 =RS. $\left(\frac{116}{100} \times 3000\right)$ =Rs.3480.
- ∴ Rate of S.P of the mixture =
- =Rs $\left[\frac{3480}{200}\right]$ perkg
- =Rs.17.40 perkg.

- 13) Pure ghee costs Rs.100 per kg. After adulterating it
- With vegetable oil costing RS.50 per kg a shopkeeper sells the mixture at the rate of Rs.96 perkg, thereby making a profit of 20% . In what ratio does he mix the two?
- Sol:
- Mean cost price =Rs. $\left[\frac{100}{120}\times 96\right]=\frac{960}{12}=Rs.80\,perkg$
- Mean price =Rs.80
- Ratio of pure ghee = Rs.100-Rs.80=Rs.20
- Ratio of vegetable oil=Rs.80-Rs.50=Rs.30
- Required ratio =30:20

- 14) A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg weight.Find the gain percent.
- Sol:
- Gain%= $\left[\frac{Error}{Truevalue - Error}\times 100\right]\%$

$$= \frac{40}{1000-40}\times 100 = \frac{40}{960}\times 100$$

$$= \frac{400}{96} = 4\frac{1}{4}$$

- 15) If the manufacture gain 10% the wholesale 15% and
- The retailer 25% ,then find the cost of production of a
- Table, the retail price of which is Rs.1265?
- Sol:
- Let the cost of production of the table be Rs.x
- Then, 125%of 115%of 110%ofx=1265

$$\frac{125}{100} \times \frac{115}{100} \times \frac{110}{100} \times x = 1265$$

$$\frac{125 \times 115 \times 11}{100000} \times x = 1265$$

$$\frac{115 \times 11}{800} \times x = 1265$$

$$\frac{115 \times 11}{800} \times x = 1265$$

$$x = \frac{1265 \times 800}{115 \times 11} = 800$$

- 16) Monika purchased a pressure cooker it $\frac{9}{10}^{th}$ of its
- Selling price and sold it at 8% more than its S.P. Find her gain percent.
- Sol:
- Let the S.P be Rs. x .Then
- C.P=Rs. $\frac{9}{10}x$
- Sold Receipt =108% of x = $\frac{108}{100} \times x = RS.\frac{27}{25}x$
- Gain=Rs. $\left[\frac{27x}{25} - \frac{9x}{10}\right]$

$$= Rs\left[\frac{108x - 90x}{100}\right] = Rs\frac{18x}{100}$$

$$\therefore Gain\% = \left(\frac{18x}{100} \times \frac{10}{9x} \times 100 \right) = 20\%$$

- 17.An article is sold at a certain price. By selling it at
- $\frac{2}{3}$ of that price one losses 10%. Find the gain percent
- At original price.
- Sol:
- Let the original be Rs.x
- S.P =Rs. $\frac{2}{3}x$ Loss=10%

$$C.P = R.S\left(\frac{100}{90} \times \frac{2}{3}x\right)$$

$$= \frac{20}{27}x$$

$$C.P = Rs.\frac{20}{27}x$$

$$Gain = Rs.\left(x - \frac{20x}{27}\right) = \frac{7x}{27}$$

$$\therefore Gain\% = \left(\frac{7x}{27} \times \frac{27}{20x} \times 100\right)\%$$

$$= 35\%$$

- 18) A tradesman sold an article at loss of 20%.If the selling price had been increased by Rs.100 there would
- Have been a gain of 5%.What was the cost price of the article?
- Sol:
- Let C.P be Rs.x

$$105\% x - 180\% x = 100$$

$$\frac{105}{100}x - \frac{80}{100}x = 100$$

$$\frac{25}{100}x = 100$$

$$\frac{x}{4} = 100$$

$$x = 400$$

- 19) A man sells an article at a profit of 25% .If he had bought it at 20% loss and sold it for Rs.10.50 less he would have gained 30% .Find the cost price of the article.
- Sol:
- Let the C.P be Rs.x
- 1st S.P =125%of x= $\frac{125}{100}x = \frac{5}{4}x$
- 2nd S.P=80%x= $\frac{80}{100}x = \frac{8}{10}x = \frac{4}{5}x$
- 2nd S.P =130% of $\frac{4}{5}x = \frac{130}{100} \times \frac{4}{5}x = \frac{26}{25}x$

$$\frac{5}{4}x - \frac{26}{25}x = 10.50$$

$$\frac{25 \times 5x - 264x}{100} = 10.50$$

$$\frac{125x - 104x}{100} = 10.50$$

$$\frac{21}{100}x = 10.50$$

$$x = \frac{10.50 \times 100}{21} = 50$$

- 20) The price of a jewel, passing through three hands ,
- rises on the whole by 65% .If the first and the second
- Sellers earned 20% and 25% . Profit respectively. Find the percentage profit earned by the third seller.
- Sol:
- Let the original price of the jewel be RS.P and let earned by the third seller be x.

$$\frac{120}{100} \times \frac{125}{100} \times \frac{100 + x}{100} \times P = \frac{165}{100} \times P$$

$$\frac{120 \times 125 \times (100 + x)}{100 \times 100 \times 100} = \frac{165}{100}$$

$$100 + x = \frac{165 \times 100 \times 100}{120 \times 125}$$

$$100 + x = 110$$

$$x = 110 - 100$$

$$x = 10\%$$

- 21) A man sold two flats for Rs.675, 958 each, on one he gain 16% while on the other he loses 16%.How much does he gain or loss in the whole transaction?
- Sol:
-

$$\therefore loss\% = \left[\frac{commonlossandgain}{10}\right]^2$$

$$= \left[\frac{16}{10}\right]^2 = \left[\frac{8}{5}\right]^2 = \frac{64}{25}\%$$

$$= 2.56\%$$

- 22) A dealer sold three –fourth of his articles at a gain of 20% and the remaining at cost price . Find the gain earned by him in the whole transaction?
- Sol: Let C.P of whole be Rs. X
- $$C.P of \frac{3}{4} th = Rs. \frac{3}{4} x$$

$$\text{Re} maning C.P of \frac{1}{4} th = x - \frac{3}{4} x = \frac{x}{4}$$

$$Totat S.P = Rs. \left[(120\% of \frac{3}{4} x) + \frac{x}{4} \right]$$

$$Total S.P = \frac{120}{100} \times \frac{3x}{4} + \frac{x}{4}$$

$$= \frac{12}{10} \times \frac{3x}{4} + \frac{x}{4}$$

$$= \frac{9x}{10} + \frac{x}{4}$$

$$= Rs. \frac{23}{20} x$$

$$Gain = Rs. (\frac{23}{20} x - x)$$

$$= Rs. \frac{3x}{20}$$

$$Gain\% = (\frac{3x}{20} \times \frac{1}{x} \times 100)\%$$

$$Gain\% = \frac{Profit}{C.P} \times 100$$

$$= 15\%$$

- 23) A man bought a horse and a carriage for Rs.3000.
- He sold the horse at a gain of 20% and the carriage at a loss of 10%,there by gaining 2% on the whole . Find the cost of the horse.
- Sol:
- Let the C.P of the horse be x and C.P of the carriage
- Be 3000-x
- Gain –Loss=Profit.
- Gain 20%ofx-loss10%of (3000-x)=Gain 2%of3000

$$\frac{20x}{100} - \frac{10}{100}(3000 - x) = \frac{2}{100} \times 3000$$

$$\frac{20x - 10(3000 - x)}{100} = \frac{2}{100} \times 3000$$

$$20x - 30000 + 10x = 2 \times 3000$$

$$30x - 30000 = 6000 + 30000$$

$$30x = 36000$$

$$= \frac{36000}{30} = 1200$$

- 24)When a producer allow 36% commission on the retail price of his product he earns a profit of 8.8%.What would be his profit percent if the commission is reduce by 24%?
- Ans:
- Let the retail price is 100 .
- Old commission 36 .
- S.P=100-36=64
- Profit=8.8%

$$C.P = RS.\left(\frac{100}{100+8.8\%}\times 64\right)$$

$$= RS.\left(\frac{100}{108.8}\times 64\right)$$

$$= RS.\frac{1000}{108.8}\times 6.4$$

$$= Rs.\frac{1000}{17}$$

- New commission =36-24=12
- New S.P=100-12=88
- Gain=S.P-C.P

$$= 88 - \frac{1000}{17}$$

$$= \frac{88 \times 17 - 1000}{17}$$

$$= \frac{496}{17}$$

$$Gain\% = \frac{\Pr ofit}{C.P} \times 100$$

$$= \frac{\frac{496}{17}}{\frac{1000}{17}} \times 100$$

$$= \frac{496}{17} \times \frac{17}{1000} \times 100 = 4.96\%$$

- 25) After getting two successive discount, a shirt with a
- List price of Rs.150 is available at Rs.105.If the second discount is 12.5%.Find the first discount?.
- Solution:
- Let the discount be x%
- Then 87.5% of (100-x)% of 150=105
- $$\frac{87.5}{100} \times \frac{100-x}{100} \times 150 = 105$$

$$\frac{87.5 \times (100-x)}{100 \times 100} \times 15 = 105$$

$$\frac{87.5\times(100-x)\times 150}{100\times 100}=105$$

$$100-x=\frac{105\times 100\times 100}{87.5\times 150}$$

$$100-x=\frac{1050000}{13125}$$

$$100-x=80$$

$$x=100-80=20$$

- Home work:
- 1.An uneducated retailer marks all his goods at 50% above the cost price and thinking that he will still make 25% profit , offers a discount of 25% on the marked price. What is his actual profit on the sales?
- 2. A retailer buys 40 pens at the marked price of 36 pens from a wholesaler . If he sells these pens giving a discount of 1% what is the profit percent?
- 3.At what percentage above the C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?
- 4.When a producer allows 36% commission on the retail price of his product, he earns a profit of 8.8%.

- What would be his profit percent if the commission is reduced by 24%
- 5.Peter purchased a machine for Rs.80,000 and spent Rs.5000 on repair and Rs.1000 on transport and sold it with 25% profit . At what price did he sell the machine?
- 6. If the selling price of 50 articles is equal to the cost price of 40 articles, then loss or gain percent is.

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UNIT-III

TIME,DISTANCE AND WORK

TIME AND DISTANCE

Example (1)

A dog takes 4 leaps for every 5 leaps of hare but 3 leaps of a dog are equal to 4 leaps of the hare.Compare their speeds.

Solution : Let the distance covered in 1 leap of dog be x and that covered in 1 leap of hare be y.

$$3x = 4y$$

$$x = \frac{4}{3}y$$

The ratio of dog and hare =4x:5y

$$= 4\frac{4}{3}y : 5y = \frac{16}{3}y : 5y$$

$$= \frac{16}{3} : 5 = 16 : 15$$

Example (2)

While covering a distance of 24 km,a man noticed that after walking for 1 hour 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance what was his speed in metres per second?

Solution: Let the speed be $x\frac{km}{hr}$

Then distance covered in 1 hr.40min

$(i.e)1\frac{2}{3}$ hrs=$\frac{5}{3}$hrs

$Distanc = Speed \times time = x \times \frac{5}{3} = \frac{5x}{3}$km

Remaining distance = $(24 - \frac{5x}{3})$ km

$\frac{5x}{3} = \frac{5}{7}(24 - \frac{5x}{3})$

Example (conti 2)

$\frac{5x}{3} = \frac{5}{7}(24 - \frac{5x}{3})$

$\frac{5x}{3} = \frac{5}{7}\frac{(72-5x)}{3}$

$7x = 72 - 5x$

$12x = 72$

$x = \frac{72}{12} = 6$

Hence speed = 6 $\frac{km}{hr} = (6 \times \frac{5}{18})\frac{m}{sec}$

=$\frac{5}{3}\frac{m}{sec}$.

Example (3)

Peter can cover a certain distance in 1hr24min.By covering two-third of the distance at 4kmph and the rest at 5mph . Find the total distance?.

Solution:

Let the total distance $x \frac{km}{p}$

$$Distance = Speed \times Time$$

$$Time = \frac{Distance}{Speed}$$

$$T_1 = \frac{\frac{2}{3}x}{4}$$

Example (conti 3)

Remaining distance in $x\frac{km}{p} = x - \frac{2}{3}x = \frac{3x-2x}{3} = \frac{1}{3}x$

$T_2 = \frac{\frac{1}{3}x}{5}$

$T_1 + T_2 = Total distance in x\frac{km}{p} = \frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5}$

$\frac{x}{6} + \frac{x}{15} = \frac{7}{5}$

$\frac{15x+6x}{6\times15} = \frac{7}{5}$

$\frac{21x}{18} = 7$

$\frac{3x}{18} = 1$

$x = 6$

Example (4)

A man travelled from the village to the post -office at the rate of 25 kmph and walked back at the rate of 4kmph;If the whole journey took 5 hours 48 minutes and the distance of the post - office from the village.

Solution: Average Speed =$(\frac{2xy}{x+y})\frac{km}{hr}$

=$\frac{2(25)\times 4}{25+4} = \frac{200}{29}\frac{km}{hr}$

Distance travelled in 5 hours 48 minutes

$(i.e)5\frac{4}{5}hrs = (\frac{200}{29} \times \frac{29}{5})km = 40km$

Distance of the post-office from the village =$(\frac{40}{2}) = 20km$.

Example (5)

An aeroplane flies along the four sides of a square at the speeds of 200,400,600 and 800 $\frac{km}{hr}$. Find the average speed of the plane around the filed.

Solution: Let each side of the square be xkm and led the average speed of the plane around the filed be $y\frac{km}{hr}$.

$$\frac{x}{200} + \frac{x}{400} + \frac{x}{600} + \frac{x}{800} = \frac{4x}{y}$$

$$\frac{12x}{2400} + \frac{6x}{2400} + \frac{4x}{2400} + \frac{3x}{2400} = \frac{4x}{y}$$

$$\frac{25x}{2400} = \frac{4x}{y}$$

$$y = \frac{2400 \times 4}{25} = 384$$

Example (6)

Walking at $\frac{5}{6}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Solution: New speed $= \frac{5}{6}$ of usual speed

$\therefore$ New time taken $\frac{6}{5}$ of the usual time.

$Distance = Speed \times time$

$Time = \frac{Distance}{Speed}$

$= \frac{1}{\frac{5}{6}} = \frac{6}{5}$ usual time

$(\frac{6}{5} of the usual time) - (usual time) = 10 min$

Example (conti 6)

$\frac{1}{5} of the usual time = 10 min$

usual time=50min

Example (7)

If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.

Solution : Let the required distance be x km.

Difference in the times taken at two speeds=7minutes+5minutes=12minutes=$12 \times \frac{1}{60} = \frac{1}{5} hr$.

$Time = \frac{Distance}{Speed}$

$\frac{x}{5} - \frac{x}{6} = \frac{1}{5}$

$\frac{6x-5x}{30} = \frac{1}{5}$

Example (conti 7)

$\frac{x}{6} = 1$

$x = 6$

Hence, the required distance is 6km.

Example (8)

A and B are two stations 390 kmph apart. A train starts from A at 10Am and travels towards B at 65kmph.Another train starts from B at 11 AM and travels towards A at 35 kmph. At what time do that meet?

Solution: Suppose they meet x hours after 10 AM,

Then
(Distance moved by first in x hrs)+(Distance moved by second in (x-1)hrs)=390

$$65x + 35(x - 1) = 390$$

Example (conti 8)

$65x + 35x - 35 = 390$

$100x = 390 + 35$

$100x = 425$

$x = \frac{425}{100} = \frac{17}{4}$

$= 4\frac{1}{4}$

so, they meet 4 hrs and 15 min, after 10 AM $(i.e)$ at 2.15 PM.

Example (9)

A goods train leaves a station at a certain time and at a fixed speed.After 6 hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the good train.

Soloution: Let the speed of the good train be x kmph.
Distance covered by good train in 10 hours= Distance covered by express in 4 hours.

$$Distance = Speed \times Time$$

$10x = 4 \times 90 = 36$
So,Speed of good train = 36kmph

Example (10)

A thief is spotted,by a policeman from a distance of 100 metres. When the policeman starts the chase,the thief also starts running. If the speed of the thief be 8 km/hr, and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?

Solution:

Relative speed of the policeman= (10-8)km/hr =2km/hr.

Time taken by policeman to cover 100m

Example (conti 10)

$= (\frac{100}{1000} \times \frac{1}{2}) = \frac{1}{20} hr$.

$Time = \frac{Distance}{Speed}$

In $\frac{1}{20} hrs$, the thief covers a distance of $(8 \times \frac{1}{20}) km = \frac{2}{5} km$

$= \frac{2 \times 1000}{5} = 400m$

Example (11)

I walk a certain distance and ride back taking a total time of 37 minutes I could walk both ways in 55 minutes. How long would it take to ride both ways?

Solution

Let the distance be x km.Then

(Time taken to walk x km)+(Time to ride x km)=37 min

(Time taken to walk $2x$ km)+ (Time taken to $2x$ km)=74 min

But time taken to walk $2x$ km=55min
Time taken to ride $2x$ km =(74-55)min=19min

TIME AND WORK

Important formula

1. If A can do a piece of work in n days, then A's 1 day's work $= \frac{1}{n}$
2. If A's 1 day's work $= \frac{1}{n}$, then A can finished the work in n days
3. If A is thrice a good a workman as B:

Ratio of work done by A and B =3:1

Ratio of times taken by A and B to finish a work =1:3

Example (1)

Worker A takes 8 hours to do job. Worker B takes 10 hours to do the same job. How long should it take both A and B, working together but independently, to do the same job?

Solution:

A's 1 hours work $= \frac{1}{8}$

B's 1 hours work $= \frac{1}{10}$

Example (conti 1)

$(A+B)$'s 1 hours's work =$(\frac{1}{8}+\frac{1}{10})=\frac{10+8}{80}=\frac{9}{40}$

$\therefore$ Both A and B will finish the work in $\frac{40}{9}$=$4\frac{4}{9}$ days.

Example (2)

A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can alone complete that work?

Solution:

$(A + B)$'s 1 day's work = $\frac{1}{4}$

A's 1 day's work =$\frac{1}{12}$

B's 1 day's work=$(\frac{1}{4} - \frac{1}{12}) = \frac{1}{6}$
Hence B alone can complete the work in 6 days.

Example (3)

A can do piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together $8\frac{2}{5}$ hours a day

Solution

A can complete the work in (7×9)=63hours.

B can complete the work in (6×7)=42 hours

Example (conti 3)

A's 1 hours work $=\frac{1}{63}$

B's 1 hours work$=\frac{1}{42}$

$(A+B)$'s 1 hour's work $=(\frac{1}{63}+\frac{1}{42})=\frac{5}{126}$

Both will finish the work in $(\frac{126}{5})$hrs

Number of days of $8\frac{2}{5}$ hrs each

$=(\frac{126}{5}\times\frac{5}{42})=3$ days.

Example (4)

A and B can do a piece of work in 18 days,B and C can do it in 24 days. A and c can do it in 36 days. In how many days will A,B and C finish it, working together and separately?

Solution:

$(A+B)'s$ 1 day's work $=\frac{1}{18} \longrightarrow (1)$

$(B+C)'s$ 1 day's work $=\frac{1}{24} \longrightarrow (2)$

$(A+C)'s$ 1 day's work$=\frac{1}{36} \longrightarrow (3)$

(1)+(2)+(3)

$2(A+B+C)'s$ 1 day's work $=\frac{1}{18}+\frac{1}{24}+\frac{1}{36}$

Example (conti4)

$(A+B+C)'s$ 1 day's work $=\frac{1}{2}[\frac{1}{18}+\frac{1}{24}+\frac{1}{36}]$

$2\lfloor\underline{18, 24, 36}$
$9\lfloor\underline{9, 12, 18}$
$2\lfloor\underline{1, 12, 2}$
$2\lfloor\underline{1, 6, 1}$
$3\lfloor\underline{1, 3, 1}$
$\lfloor\underline{1, 1, 1}$

Example (conti4)

$LCM = 2 \times 9 \times 2 \times 2 \times 3 = 24 \times 9 = 8 \times 3 \times 9 = 216$

$(A + B + C)'s$ 1 day's work = $\frac{1}{2}[\frac{12+9+6}{216}]$

$= \frac{1}{2}[\frac{27}{216}] = \frac{1}{2}(\frac{1}{8}) = \frac{1}{16}$

$\therefore (A + B + C)'s$ 1 day's work=$\frac{1}{16}$

Thus A,B and C together can finish the work in 16 days,

Now $A's$ 1 day's work =[(A+B+C)'s day's work]-[(B+C)'s 1 day's work]

Example (conti 4)

$= (\frac{1}{16} - \frac{1}{24}) = \frac{24-16}{16\times24}$

$= \frac{8}{16\times24} = \frac{1}{48}$

A alone can finish the work in 48 days

Similarly $B's$ 1 day's work

Now $B's$ 1 day's work =[(A+B+C)'s day's work]-[(A+C)'s 1 day's work]

$= (\frac{1}{16} - \frac{1}{36}) = \frac{36-16}{36\times16}$

$= \frac{20}{36\times16} = \frac{5}{144}$

Example (conti4)

B alone can finish the work in $\frac{144}{5} = 28\frac{4}{5}$ days

And $C's$ alone 1 day's work

Now $C's$ 1 day's work =[(A+B+C)'s day's work]-[(A+B)'s 1 day's work]

$= (\frac{1}{16} - \frac{1}{18}) = \frac{18-16}{16\times18}$

$= \frac{2}{16\times18} = \frac{1}{144}$ $\therefore$ C alone can finish the work in144 days

Example (5)

A is twice as good a work man as B and together they finish a piece of work in 18 days.In how many days will A alone finish the work?

Solution

(A's 1 day's work):(B's 1 day's work)=2:1

(A+B)'s 1 day's work $=\frac{1}{18}$

Divide $\frac{1}{18}$ in the ratio 2:1

$A's$ 1 day's work $=\frac{1}{18} \times \frac{2}{3} = \frac{1}{27}$

Example (6)

A can do a certain job in 12 days. B is 60 percentage more efficient than A. How many days does B alone take do the same job?

Solution:

Ratio of the taken by A and B =160:100

Suppose B alone takes x days to do the job:

160:100=12:x

$$\frac{160}{100} = \frac{12}{x}$$

Example (conti6)

$\frac{8}{5} = \frac{12}{x}$

$\frac{2}{5} = \frac{3}{x}$

$x = \frac{15}{2} = 7\frac{1}{2}$

Example (7)

A can do a piece of work in 80 days the works at it for 10 days and then B alone finishes the remaining work in 42 days.In how much time will A and B working together finish the work?

Solution:

Work done by A in 10 days = $(\frac{1}{80} \times 10 = \frac{1}{8})$

Remaining work =$(1 - \frac{1}{8}) = \frac{8-1}{8} = \frac{7}{8}$

Now ,$\frac{7}{8}$ work is done by B in 42 says

Whole work will be done by B in $(42 \times \frac{8}{7}) = 48$ days

$\therefore A's 1 day's work = \frac{1}{80}$

Example (conti 7)

$B's1day'swork = \frac{1}{48}$

$\therefore (A+B)'s1day'swork$

$= (\frac{1}{80} + \frac{1}{48}) = \frac{48\times80}{80\times48}$

$= \frac{128}{80\times48} = \frac{1}{30}$

Example (8)

A and B undertake to do a piece of work for Rs.600.A alone can do it in 6 days while B alone can do it in 8 days with the help of c, they finish it in 3 days. Find the share of each

Solution:

$C's$ 1 day's work $=\frac{1}{3} - (\frac{1}{6} + \frac{1}{8})$

$$= \frac{1}{3} - (\frac{8+6}{48}) = \frac{1}{3} - \frac{14}{48} = \frac{16-14}{48} = \frac{2}{48} = \frac{1}{24}$$

$\therefore$ A:B:C=Ratio of their 1 day's work $= \frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1$

Example (conti 8)

$A's$ share =Rs.$(600 \times \frac{4}{8}) = \frac{2400}{8} = Rs.300$

$B's$ share =Rs.$(600 \times \frac{3}{8}) = \frac{1800}{8} = Rs.225$

$C's$ share =$Rs.(600 \times \frac{1}{8}) = RS.75$

Example (9)

A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately. A beginning, in how many day's the work will be completed?

Solution:

$(A+B)'s$ 2 day's work $=(\frac{1}{9}+\frac{1}{12})$

$=\frac{12+9}{9\times12}=\frac{21}{9\times12}=\frac{7}{36}$

Work done in 5 pairs of days

Example (conti9)

$= 5 \times \frac{7}{36} = \frac{35}{36}$

Remaining work =$(1 - \frac{35}{36}) = \frac{1}{36}$

on 11^{th} day, it is $A's$ turn.$\frac{1}{9}$ work is done by him in 1 day. $\frac{1}{36}$ work is done by him is$(9 \times \frac{1}{36}) = \frac{1}{4}$

$\therefore$ Total time= $(10 + \frac{1}{4})$days=$10\frac{1}{4}$

Example (10)

45 men can complete a work in 16 days. Six days after they started working,30 more men joined them. How many days will they now take to complete the remaining work?

Solution:

45×16 men can complete the work in 1 day.

$\therefore$ 1 men's 1 day's work=$\frac{1}{720}$

45 men's 6 day's work =$\frac{1}{16} \times 6 = \frac{3}{8}$

Remaining work =$1 - \frac{3}{8} = \frac{5}{8}$

75 men's 1 day's work =$\frac{75}{720} = \frac{5}{48}$

Example (conti 10)

Now, $\frac{5}{48}$ work is done by them in 1 day.

$\therefore \frac{5}{8}$ work is done by them in $(\frac{48}{5} \times \frac{5}{8})$=6days

Example (11)

2 men 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days.In how many days can 2 men and 1 boy do the work?

Solution

Let 1 man's 1 day work $=x$

1 boy's 1 day work $=y$

$2x + 3y = \frac{1}{10} \longrightarrow (1)$

$3x + 2y = \frac{1}{8} \longrightarrow (2)$

Example (conti 11)

$(1) \times 3 - (2) \times 2$

$$\begin{array}{l} 6x + 9y = \frac{3}{10} \\ -6x - 4y = -\frac{2}{8} \\ \hline 0x + 5y = \frac{3}{10} - \frac{2}{8} = \frac{24-20}{80} \end{array}$$

$$5y = \frac{4}{80} = \frac{1}{20}$$

$$y = \frac{1}{100}$$

y value sub in equation (1)

Example (conti 11)

$2x + 3\frac{1}{100} = \frac{1}{10}$

$2x = \frac{1}{10} - \frac{3}{100}$

$2x = \frac{10-3}{100}$

$x = \frac{7}{200}$

(2men+1boy)'s 1 day's work

$=2 \times \frac{7}{200} + 1 \times \frac{1}{100} = \frac{14}{200} + \frac{2}{200} = \frac{16}{200}$

$= \frac{2}{25}$

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$

Thank You

PROBLEM ON AGES

- 1.Rajeev’s age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev?
- Ans:
- Rajeev’s age be x years.
- Rajeev’s age after 15 years =(x+15)years.
- Rajeev’s age 5 years back = (x-5)years
- X+15=5(x-5)
- X+15=5x-25

- $5x-x=25+15=40$
- $4x=40$
- $X=10$
- 2.The ages two persons differ by 16 years. If 6 years ago, elder one be 3 times as old as the younger one, find their present age?
- Ans :
- Let the younger person be x years.
- The age of the older person=x+16

- $3(x-6)=(x+16-6)$
- $3x-18=x+10$
- $3x-x=10+18=28$
- $2X=28$
- $X=28/2=14$
- So other person age is $x+16=14+16=30$.
- 3.The product of the ages of Ankit and Nikita is 240.If twice the age of Nikita is more than Ankit’s age by 4 years, what is Nikita’s age?

- Ans:
- Let Ankit age be x, and Nikita age be 240/x.
- $2(240/x)-x=4$
- $480/x-x=4$
- $480-x^2=4x$
- $x^2+4X-480=0$
- $(x+24)(x-20)=0$
- $X=20.$
- –

- 4.The present age of a father is 3 years more than three times the age of his son.
- Three years hence , father ‘s the age will be 10 years more than twice the age of the son . Find the present age of the father.
- Ans:
- Let the son’s age be x and father’s age be 3x+3
- $(3x+3+3)=2(x+3)+10$
- $3x+6=2x+6+10$
- $3x-2x=10 \Rightarrow x=10.$

- 5.Rohit was 4 times old as his son 8 years ago. After 8 years Rohit will be twice as old as his son. What are their present age?
- Ans:
- Let the son’s age 8 years ago be x years.
- Then Rohit ‘s age 8 years ago=4x
- Rohit’s age after 8 years =(4x+8)+8=4x+16
- Son’s age after the 8 years =x+16
- 2(x+16)=4x+16
- 2x+32=4x+16
- 4x-2x=32-16
- 2x=16
- X=8
- Hence son’s age =x+8=8+8=16 years
- Rohit’s Present age =4x+8=4(8)+8 =40 Years

- 6.One year ago , the ratio of Gaurav’s and Sachin’s age was 6:7 respectively Four years hence , this ratio would become 7:8 .How old is sachin?
- Sol:
- Let Gaurav’s and sachin’s ages one year ago be 6x and 7x years respective.
- Gaurav’s age 4 years hence=(6x+1)+4
- =6x+5
- Sachin’s age 4 years hence =(7x+1)+4
- =7x+5

$$\frac{6x+5}{7x+5}=\frac{7}{8}$$

- 8(6x+5)=7(7x+5)
- 48x+40=49x+35
- 49x-48x=40-35
- X=5
- 7.Abhay’s age after six years, will be three-seventh of his father’s age.Ten years age the ratio of their age was 1:5.What is Abhay’s father’s age presnt?
- Ans:
- Let the age of Abhay and his father 10 years ago be x and 5x years respectively.

- Then Abhay's age after 6 years =(x+10)+6=x+16 years.
- Father's age after 6 years=(5x+10)+6
- =5x+16years
- X+16=3/7 (5x+16)
- 7(x+16)=3(5x+16)
- 7x+112=15x+48
- 15x-7x=112-48
- 8x=64
- X=8
- Hence Abhay's father's persent age=5(8)+10=50 years.

A Brief History of Mathematics

A Brief History of Mathematics

What is mathematics?

What do mathematicians do?

A Brief History of Mathematics

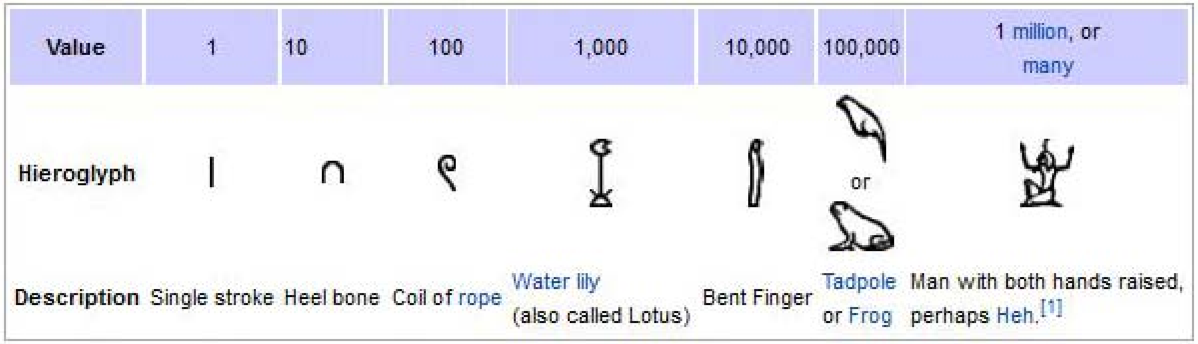
What is mathematics?

What do mathematicians do?

http://www.sfu.ca/~rpyke/presentations.html

A Brief History of Mathematics

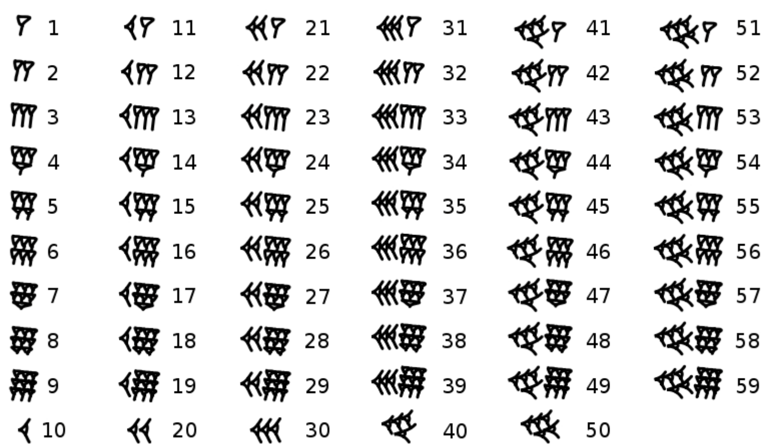
- Egypt; 3000B.C.
  - Positional number system, base 10
  - Addition, multiplication, division. Fractions.
  - Complicated formalism; limited algebra.
  - Only perfect squares (no irrational numbers).
  - Area of circle; $(8D/9)^2 \rightarrow \prod=3.1605$. Volume of pyramid.



Value	1	10	100	1,000	10,000	100,000	1 million, or many
Hieroglyph						or	
Description	Single stroke	Heel bone	Coil of rope	Water lily (also called Lotus)	Bent Finger	Tadpole or Frog	Man with both hands raised, perhaps Heh.[1]

A Brief History of Mathematics

- Babylon; 1700-300B.C.
  - Positional number system (base 60; sexagesimal)
  - Addition, multiplication, division. Fractions.
  - Solved systems of equations with many unknowns
  - No negative numbers. No geometry.
  - Squares, cubes, square roots, cube roots
  - Solve quadratic equations (but no quadratic formula)
  - Uses: Building, planning, selling, astronomy (later)



A Brief History of Mathematics

- Greece; 600B.C. – 600A.D.   Papyrus created!
  - Pythagoras; mathematics as abstract concepts, properties of numbers, irrationality of $\sqrt{2}$, Pythagorean Theorem $a^2+b^2=c^2$, geometric areas
  - Zeno paradoxes; infinite sum of numbers is finite!
  - Constructions with ruler and compass; ‘Squaring the circle’, ‘Doubling the cube’, ‘Trisecting the angle’
  - Plato; plane and solid geometry

A Brief History of Mathematics

- Greece; 600B.C. – 600A.D.

Aristotle; mathematics and the physical world (astronomy, geography, mechanics), mathematical formalism (definitions, axioms, proofs via construction)

- Euclid; *Elements* – 13 books. Geometry, algebra, theory of numbers (prime and composite numbers, irrationals), method of exhaustion (calculus!), Euclid's Algorithm for finding greatest common divisor, proof that there are infinitely many prime numbers, **Fundamental Theorem of Arithmetic** (all integers can be written as a product of prime numbers)
- Apollonius; conic sections
- Archimedes; surface area and volume, centre of gravity, hydrostatics
- Hipparchus and Ptolemy; Trigonometry (circle has 360°, sin, cos, tan; $\sin^2 + \cos^2 = 1$), the *Almagest* (astronomy; spherical trigonometry).
- Diophantus; introduction of symbolism in algebra, solves polynomial equations

Some mathematical facts known to the ancient Greeks

- There are infinitely many prime numbers:

(prime P: only factors of P are 1 and P)

Some mathematical facts known to the ancient Greeks

- There are infinitely many prime numbers:

  (prime P: only factors of P are 1 and P)

  - Suppose not. So there is a largest prime; $\hat{P}$.

    Let M = 2•3•5•7•11•13•17• • • • $\hat{P}$ (product of all primes)

    Note that none of these primes can divide M+1 (remainder is 1).

    But M+1 = $q_1 q_2 q_3 \bullet\bullet\bullet q_n$, product of primes, by the <u>Fundamental Theorem of Arithmetic</u>. What are these primes q? So there must be more primes than the ones factoring M.

Some mathematical facts known to the ancient Greeks

- $\sqrt{2}$ is not a rational number:

Some mathematical facts known to the ancient Greeks

- √2 is not a rational number:

Suppose it was rational; then √2 = a/b, a&b integers.

We can assume that ‘a’ and ‘b’ have no common factors.

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Hence a^2 is an even integer. But 'a' cannot be an odd integer (because odd•odd is an odd integer) and so it must be an even integer; $\boxed{a = 2k}$

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And so $(2k)^2 = 2b^2 \rightarrow 2k^2 = b^2 \rightarrow$ b is also even!

But this contradicts the assumption that ‘a’ and ‘b’ have no common factors.

A few important problems in the development of mathematics

A few important problems in the development of mathematics

<u>Solving polynomial equations (roots of equations)</u>

A few important problems in the development of mathematics

Solving polynomial equations (roots of equations)

– Linear; $ax + b = 0$ → $x = -b/a \quad (a \neq 0)$

A few important problems in the development of mathematics

Solving polynomial equations (roots of equations)

– Quadratic; 'easy' case $ax^2 + b = 0 \rightarrow$ $x = \pm \sqrt{-b/a}$

A few important problems in the development of mathematics

Solving polynomial equations (roots of equations)

– Quadratic; ‘easy’ case $ax^2 + b = 0 \rightarrow x = \pm \sqrt{-b/a}$

general case; $ax^2 + bx + c = 0$;

→ complete the square;

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

*quadratic formula; **formula** for the solution*

Known since ancient times

A few important problems in the development of mathematics

Solving other polynomial equations

– Cubic ; $x^3 + bx^2 + cx + d = 0$

– Quartic; $x^4 + ax^3 + bx^2 + cx + d = 0$

General solutions discovered around 1550;

a **formula** that gives you the solutions in terms of a,b,c,d and works for **all** such polynomials using **only roots**

$$\sqrt{\cdots}, \quad \sqrt[3]{\cdots}, \quad \sqrt[4]{\cdots}$$

A few important problems in the development of mathematics

Solving polynomial equations

– Quintic; $x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$

A few important problems in the development of mathematics

Solving polynomial equations

– Quintic; $x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$

Is there a formula, with only a,b,c,d,e in it, that gives you the solutions (roots) using only square roots, cube roots, fourth and fifth roots?

A few important problems in the development of mathematics

Solving polynomial equations

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Many tries. Suspected not possible in 1700's.

A few important problems in the development of mathematics

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Liouville announces some reasons why; 1843.

A few important problems in the development of mathematics

<u>Solving polynomial equations</u>

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Many tries. Suspected not possible in 1700's.

Liouville announces some reasons why; 1843.

Galois solves problem around same time →

ushers in new ideas into algebra; <u>*Galois Theory*</u>

Now we know <u>***why***</u> for quintic (and higher) polynomials there is <u>no formula</u> for the roots and that works for *all* polynomials

A few important problems in the development of mathematics

The development of calculus (1600’s)

A few important problems in the development of mathematics

The development of calculus (1600's)

Motivated by 4 problems;

1. Instantaneous velocity of accelerating object
2. Slope of a curve (slope of tangent line)
3. Maximum and minimum of functions
4. Length of (non-straight) curves (e.g., circumference of an ellipse?

A few important problems in the development of mathematics

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1. Instantaneous velocity of accelerating object
2. Slope of a curve (slope of tangent line)
3. Maximum and minimum of functions
4. Length of (non-straight) curves (e.g., circumference of an ellipse? $L = 4a\int_0^{\pi/2}\sqrt{1-\frac{a^2-b^2}{a^2}\sin^2\phi\;d\phi}$)

A few important problems in the development of mathematics

<u>The development of calculus</u> 1600's

Using calculus, Newton explained (in the *Principia*);

- why tides occur
- why the shapes of planetary orbits are conic sections (ellipses, parabolas, and hyperbolas)
- Kepler's 3 Laws of planetary motion
- shape of a rotating body of fluid
- etc, etc, etc

A few important problems in the development of mathematics

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- etc, etc, etc

and then .....

A few important problems in the development of mathematics

The development of calculus 1600's

The discovery of Neptune on paper! (1846)

(Celestial Mechanics)

(Uranus 'accidentally' discovered by telescope; William Herschel 1781)

A few important problems in the development of mathematics

The notion of **infinity** 20th Century

$$\infty \quad \text{or} \quad \aleph_0$$

A few important problems in the development of mathematics

The notion of **infinity** 20th Century

$$\infty \quad \text{or} \quad \aleph_0$$

"Aleph"

Georg Cantor; mathematician 1845 - 1918

A few important problems in the development of mathematics

The notion of **infinity** 20^{th} Century

$$\infty \quad \text{or} \quad \aleph_0$$

How many integers are there?

∞ "=" {1,2,3,....} The 'usual' infinity is the whole set of **natural numbers** (counting numbers) .

A few important problems in the development of mathematics

The notion of **infinity** 20th Century

$$\infty \quad \text{or} \quad \aleph_0$$

How many integers are there?

∞ "=" {1,2,3,....} The 'usual' infinity is the whole set of **natural numbers** (counting numbers) .

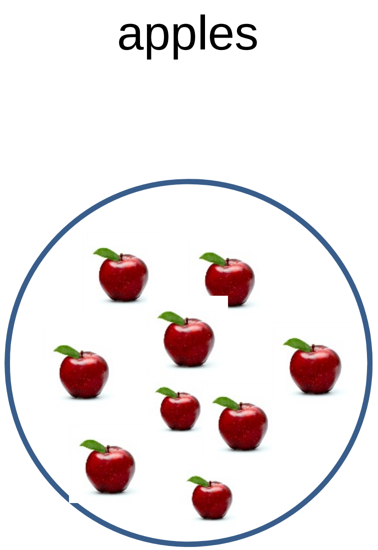
Remarkably, this is the same 'size' as all the integers (positive and negative), and the same 'size' as all the rational numbers!

- How do we count?

- How do we count?

  By ‘labeling’ the objects with counting numbers!

- How do we count?



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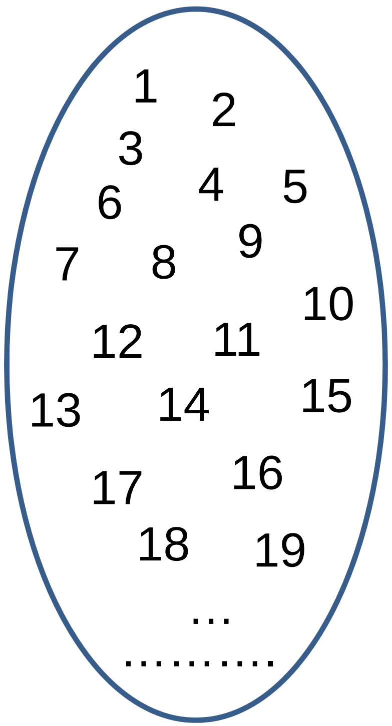
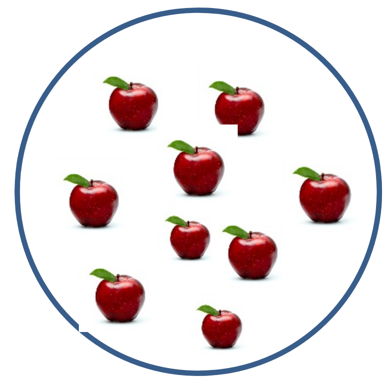
natural numbers



apples



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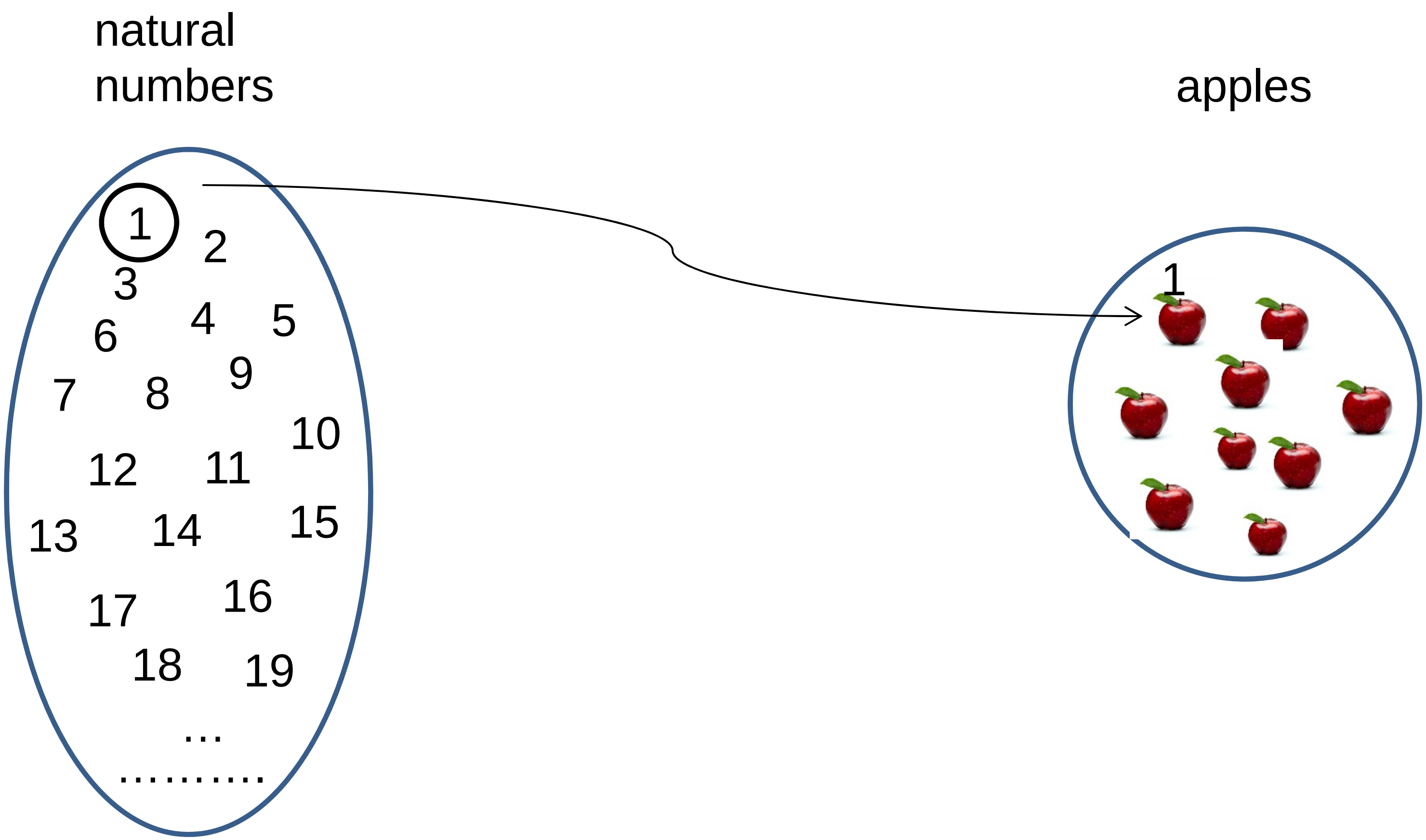

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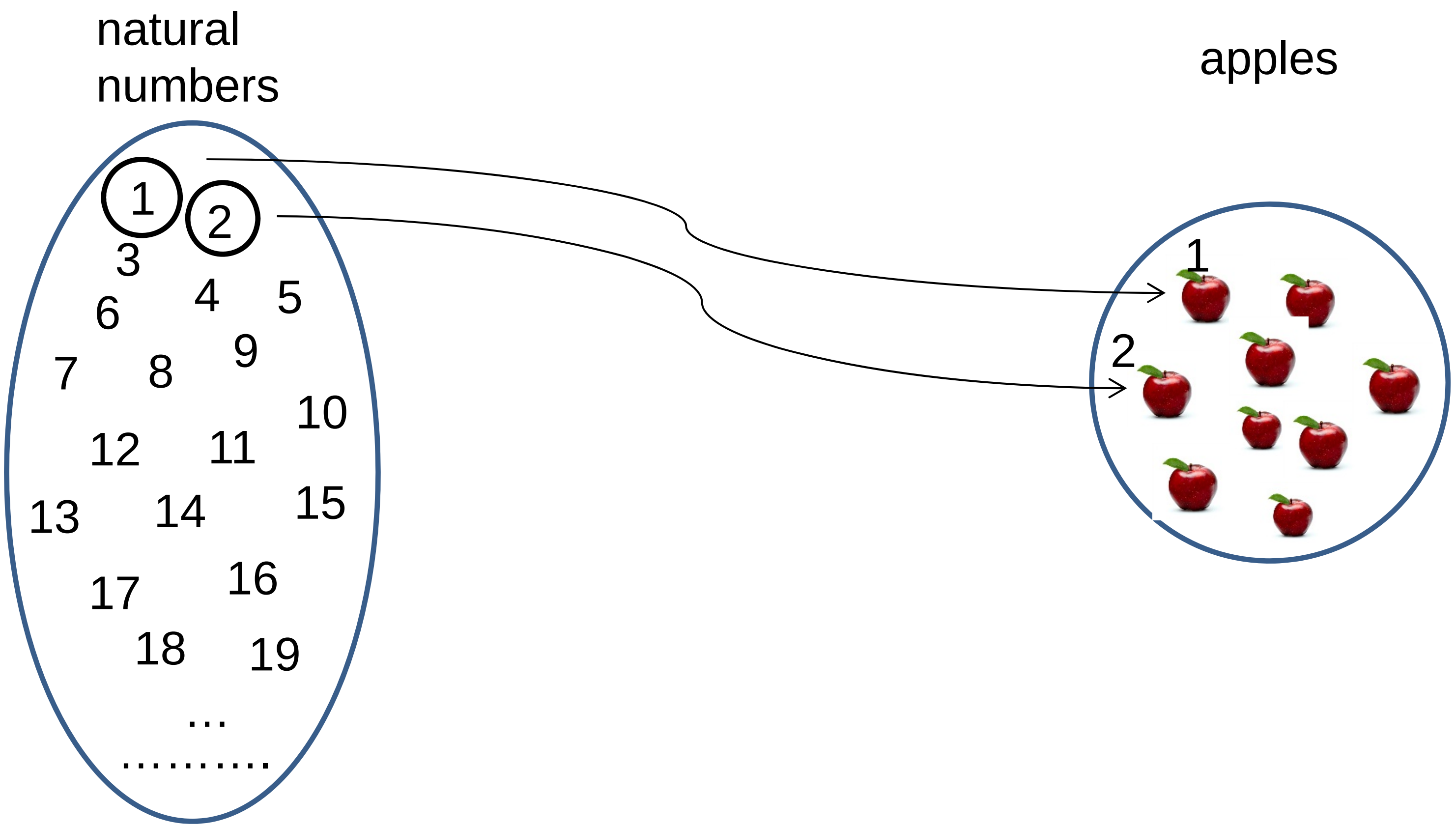

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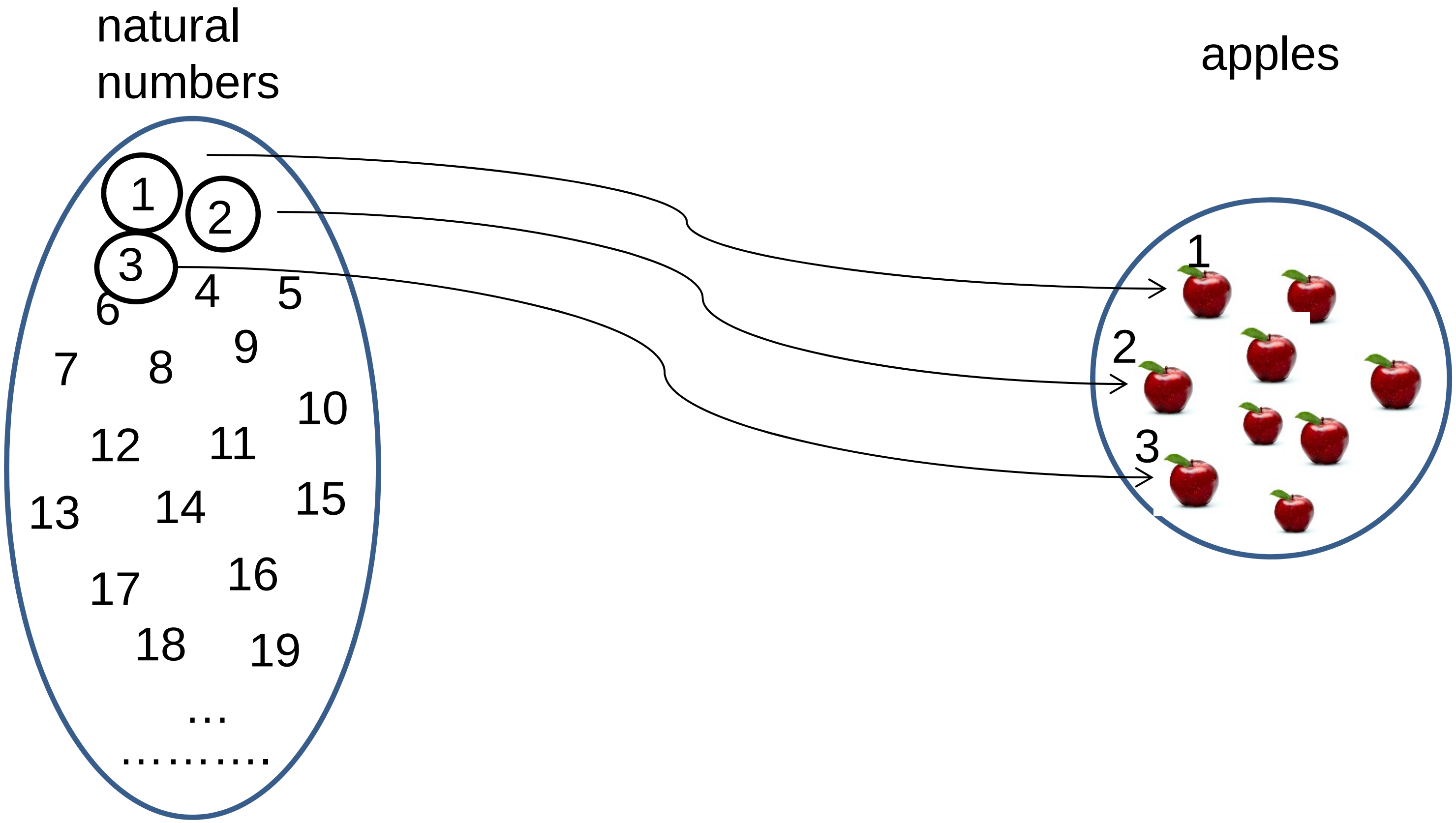

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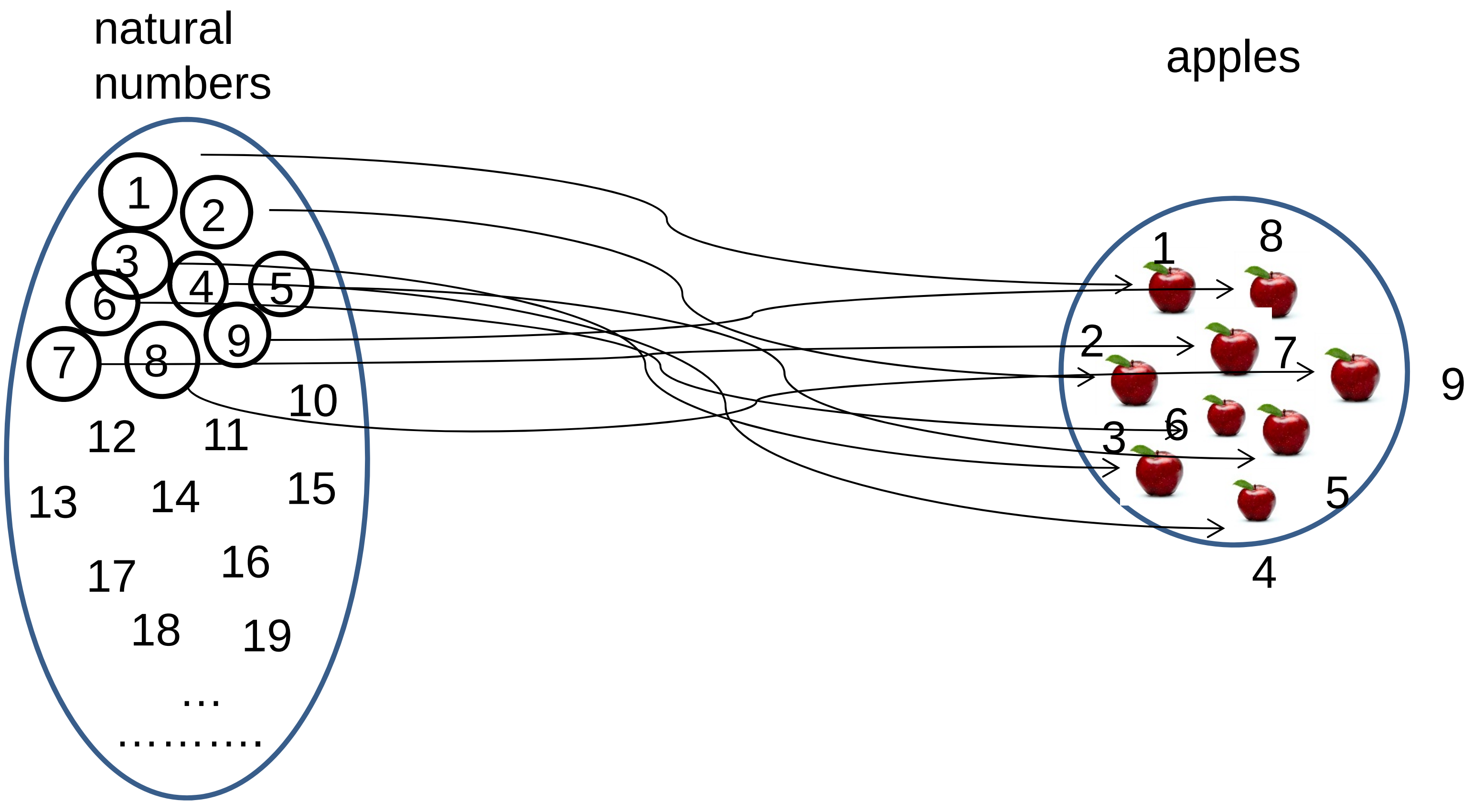

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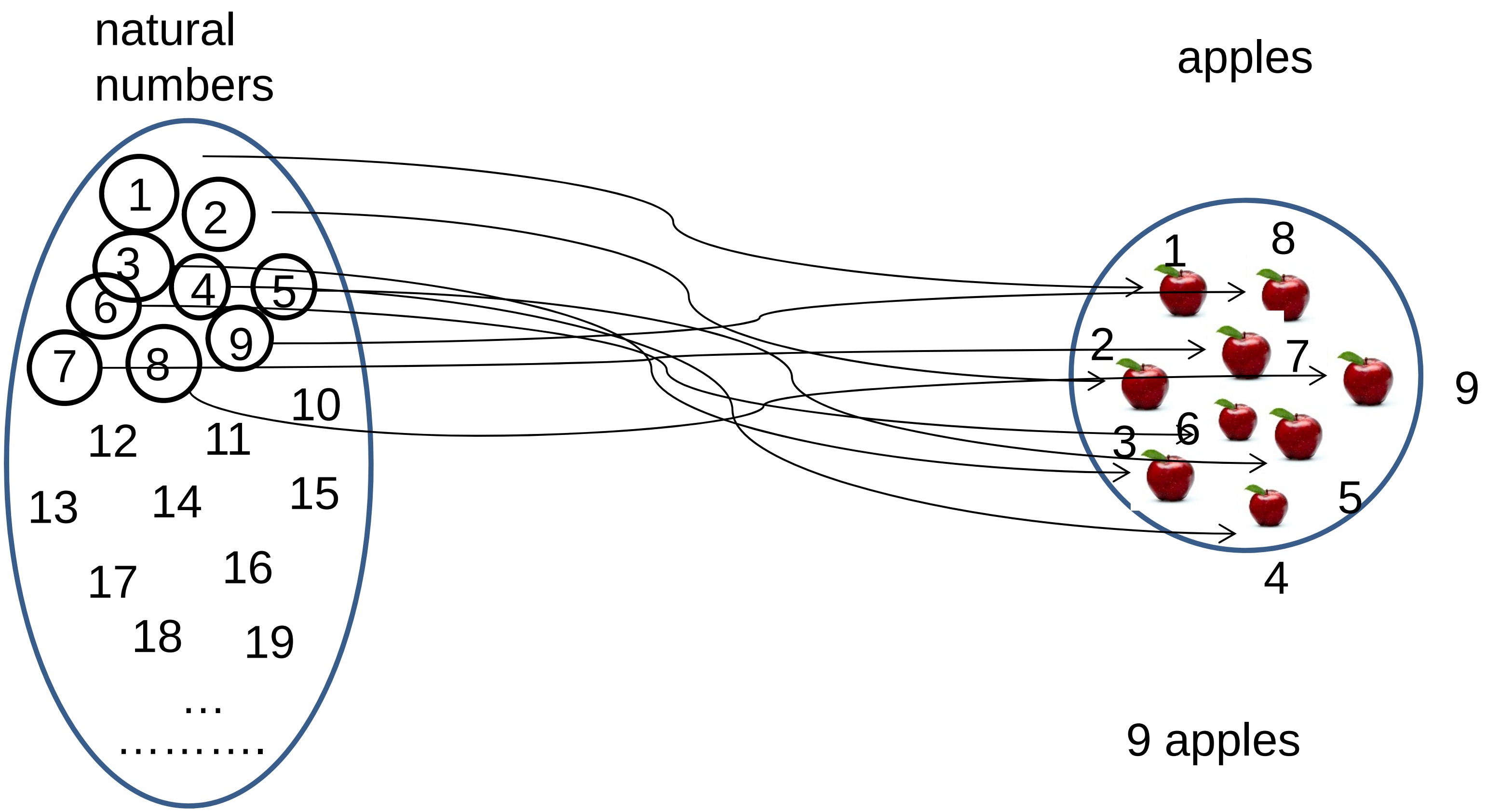

Set of integers: $\mathcal{Z}$ = { . . . -3, -2, -1, 0, 1, 2, 3, . . . }

<u>There are (only) ∞ many integers;</u>

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<u>There are (only) ∞ many integers;</u>

Counting numbers (natural numbers);

1 2 3 4 5 6 7 8 9 10 . . . .

Integers;

. . . -3 -2 -1 0 1 2 3 . . .

Set of integers: $\mathcal{Z}$ = { . . . -3, -2, -1, 0, 1, 2, 3, . . . }

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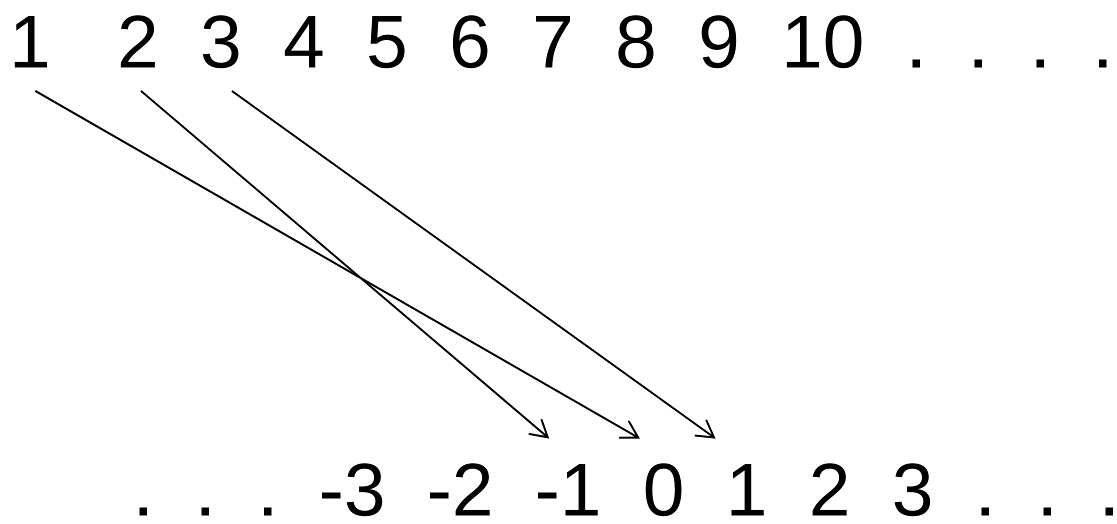

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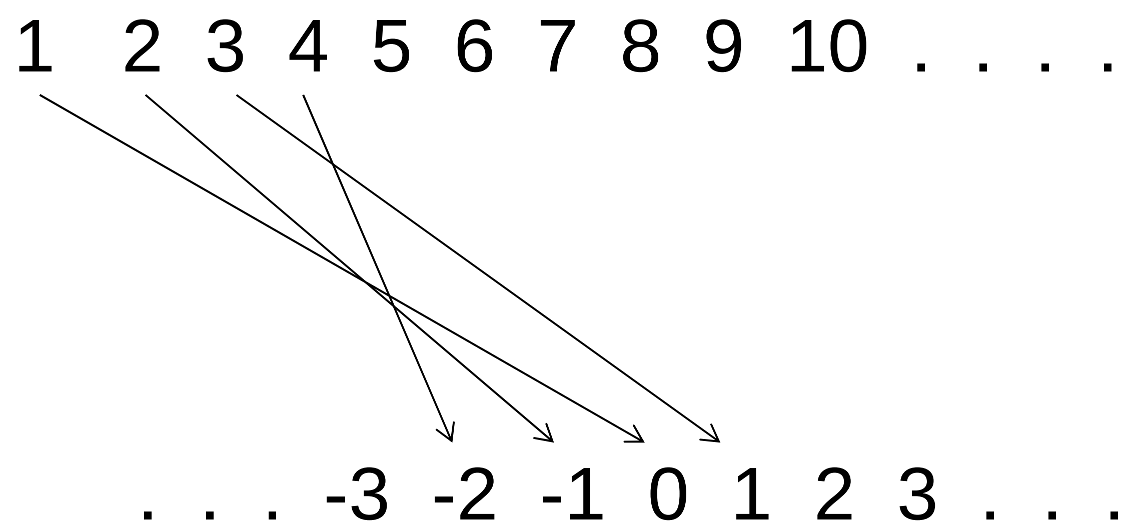

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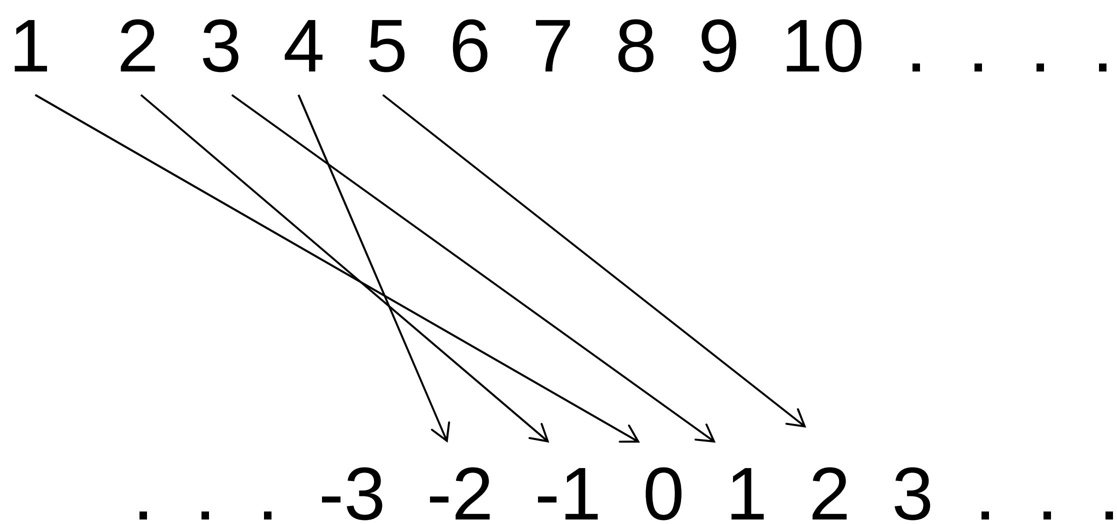

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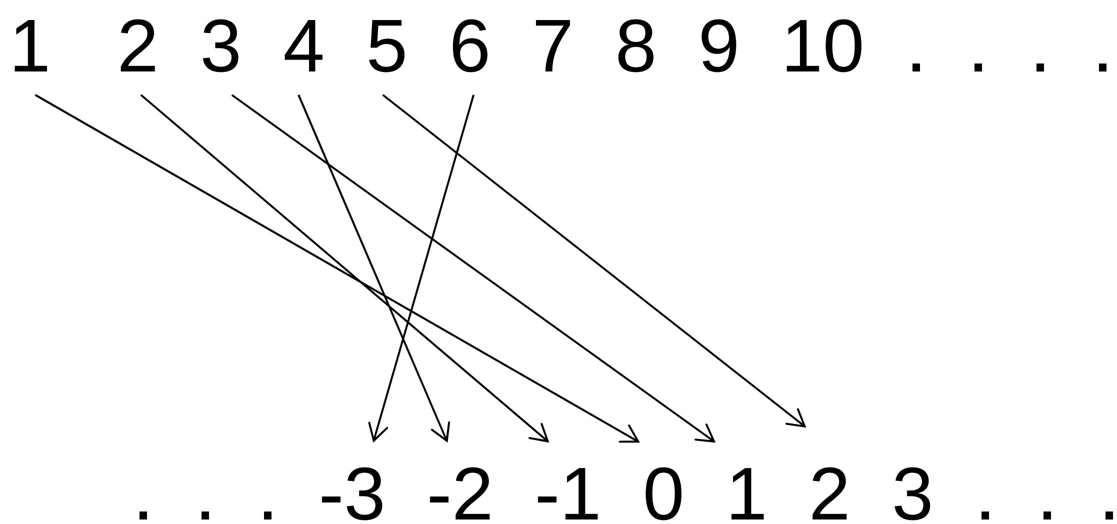

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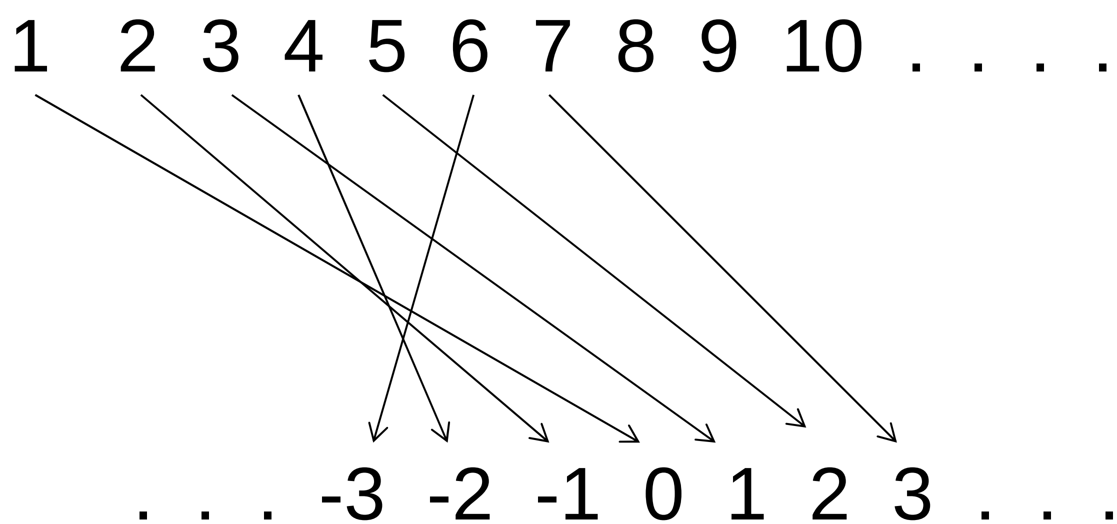

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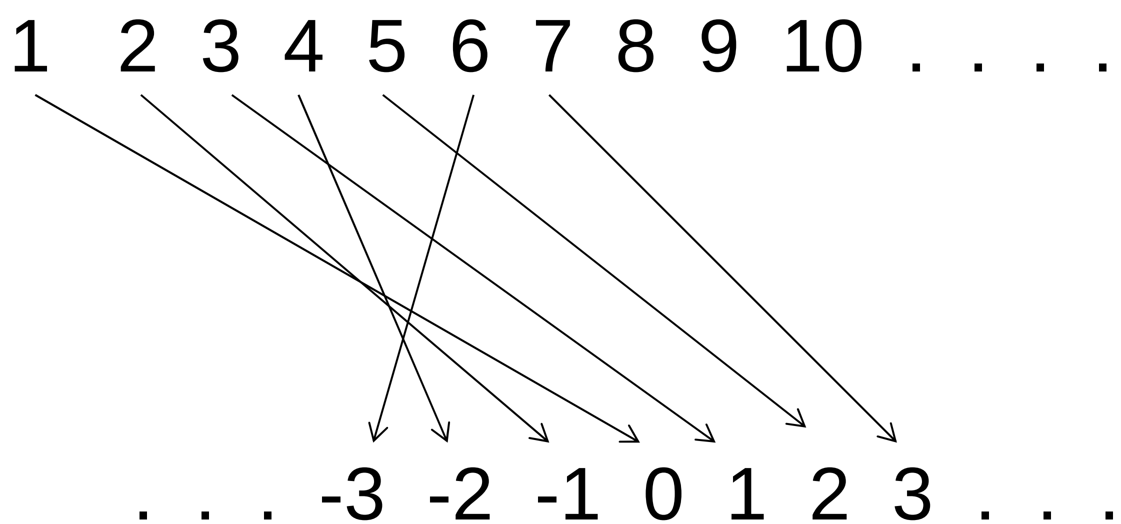

We can count all the integers this way!

A few important problems in the development of mathematics

The notion of infinity 20th Century

Algebra with infinity;

A few important problems in the development of mathematics

The notion of infinity 20th Century

Algebra with infinity;

- $\infty + 1 = \infty$, $\infty + 100 = \infty$, etc
- $\infty \bullet \infty = \infty$
- $\infty^{\infty} = \infty$

A few important problems in the development of mathematics

The notion of infinity 20th Century

Algebra with infinity;

- $\infty + 1 = \infty, \quad \infty + 100 = \infty,$ etc
- $\infty \bullet \infty = \infty$
- $\infty^{\infty} = \infty$

Is there anything bigger than ∞ ?

A few important problems in the development of mathematics

The notion of infinity 20th Century

There are (only) ∞ many rational numbers . . .

A few important problems in the development of mathematics

The notion of infinity 20th Century

There are (only) ∞ many rational numbers . . .

Is there anything larger ? ………..

A few important problems in the development of mathematics

<u>The notion of infinity</u> 20th Century

There are (only) ∞ many rational numbers . . .

Is there anything larger ? ………..

Decimal representation of a number in [0, 1];

0. $d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7\ d_8\ d_9$ ••• each d is 0,1,2,…8, or 9

e.g., 0.2154960012…….. Here, d_1=2, d_2=1, d_3=5, etc.

A few important problems in the development of mathematics

The notion of infinity 20th Century

How many *real numbers* are there?? Let's count;

1   $0.\, d_1^1\, d_2^1\, d_3^1\, d_4^1\, d_5^1\, d_6^1\, d_7^1\, d_8^1\, d_9^1 \bullet\bullet\bullet$

2   $0.\, d_1^2\, d_2^2\, d_3^2\, d_4^2\, d_5^2\, d_6^2\, d_7^2\, d_8^2\, d_9^2 \bullet\bullet\bullet$

3   $0.\, d_1^3\, d_2^3\, d_3^3\, d_4^3\, d_5^3\, d_6^3\, d_7^3\, d_8^3\, d_9^3 \bullet\bullet\bullet$

•

•

•

(decimal representation of numbers in [0, 1])

A few important problems in the development of mathematics

The notion of infinity 20th Century

1 $0.d_1^1 d_2^1 d_3^1 d_4^1 d_5^1 d_6^1 d_7^1 d_8^1 d_9^1 \cdots$

2 $0.d_1^2 d_2^2 d_3^2 d_4^2 d_5^2 d_6^2 d_7^2 d_8^2 d_9^2 \cdots$

3 $0.d_1^3 d_2^3 d_3^3 d_4^3 d_5^3 d_6^3 d_7^3 d_8^3 d_9^3 \cdots$

•

•

A few important problems in the development of mathematics

<u>The notion of infinity</u> 20th Century

1 $0.\, d_1^1\, d_2^1\, d_3^1\, d_4^1\, d_5^1\, d_6^1\, d_7^1\, d_8^1\, d_9^1 \cdots$

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3 $0.\, d_1^3\, d_2^3\, d_3^3\, d_4^3\, d_5^3\, d_6^3\, d_7^3\, d_8^3\, d_9^3 \cdots$

•

•

Here's a number;

$x = 0.\, x_1\, x_2\, x_3\, x_4\, x_5\, x_6\, x_7\, x_8\, x_9 \cdots$ where

$x_1 \neq d_1^1,\ x_2 \neq d_2^2,\ x_3 \neq d_3^3,\ x_4 \neq d_4^4,\ x_5 \neq d_5^5,\ \cdots$

A few important problems in the development of mathematics

The notion of infinity 20th Century

1 $0.\, d_1^1\, d_2^1\, d_3^1\, d_4^1\, d_5^1\, d_6^1\, d_7^1\, d_8^1\, d_9^1 \cdots$

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$x_1 \neq d_1^1$, $x_2 \neq d_2^2$, $x_3 \neq d_3^3$, $x_4 \neq d_4^4$, $x_5 \neq d_5^5$, $\cdots$

Check: This x is not in our list above!!

A few important problems in the development of mathematics

1 $0.\ d_1^1\ d_2^1\ d_3^1\ d_4^1\ d_5^1\ d_6^1\ d_7^1\ d_8^1\ d_9^1 \bullet\bullet\bullet$

2 $0.\ d_1^2\ d_2^2\ d_3^2\ d_4^2\ d_5^2\ d_6^2\ d_7^2\ d_8^2\ d_9^2 \bullet\bullet\bullet$

3 $0.\ d_1^3\ d_2^3\ d_3^3\ d_4^3\ d_5^3\ d_6^3\ d_7^3\ d_8^3\ d_9^3 \bullet\bullet\bullet$

•

$x = 0.\ x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9 \bullet\bullet\bullet$ where

$x_1 \neq d_1^1$, $x_2 \neq d_2^2$, $x_3 \neq d_3^3$, $x_4 \neq d_4^4$, $x_5 \neq d_5^5$, $\bullet\bullet\bullet$ (note there are *many* such choices for x)

x is not in our list!!

So there are more real numbers than ∞

A few important problems in the development of mathematics

1 $0.\ d_1^1\ d_2^1\ d_3^1\ d_4^1\ d_5^1\ d_6^1\ d_7^1\ d_8^1\ d_9^1 \bullet\bullet\bullet$

2 $0.\ d_1^2\ d_2^2\ d_3^2\ d_4^2\ d_5^2\ d_6^2\ d_7^2\ d_8^2\ d_9^2 \bullet\bullet\bullet$

3 $0.\ d_1^3\ d_2^3\ d_3^3\ d_4^3\ d_5^3\ d_6^3\ d_7^3\ d_8^3\ d_9^3 \bullet\bullet\bullet$

•

$x = 0.\ x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9 \bullet\bullet\bullet$ where

$x_1 \neq d_1^1$, $x_2 \neq d_2^2$, $x_3 \neq d_3^3$, $x_4 \neq d_4^4$, $x_5 \neq d_5^5$, $\bullet\bullet\bullet$ (note there are *many* such choices for x)

x is not in our list!!

So there are more real numbers than ∞

How many real numbers are there? . . . .

A few important problems in the development of mathematics

<u>The notion of infinity</u> 20th Century

The **Continuum Hypothesis** (1900);

The size of the real numbers is the 'next' infinity after ∞;

$$|\mathcal{Z}| = \infty = \boxed{\aleph_0 < |\mathcal{R}|} = \aleph_1 < \aleph_2 < \cdots$$

A few important problems in the development of mathematics

The notion of infinity 20^{th} Century

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Proof?

A few important problems in the development of mathematics

The notion of infinity 20^{th} Century

The **Continuum Hypothesis** (1900);

The size of the real numbers is the 'next' infinity after ∞;

$$|\mathcal{Z}| = \infty = \boxed{\aleph_0 < |\mathcal{R}|} = \aleph_1 < \aleph_2 < \cdots$$

Proof? In fact, *this statement cannot be proved to be true nor can it be proved to be false*! (1963) In other words, assuming it is true or assuming it is false will not get you into trouble ~ (See Gödel's *Incompleteness Theorem*, 1931)

Some important questions in modern mathematics

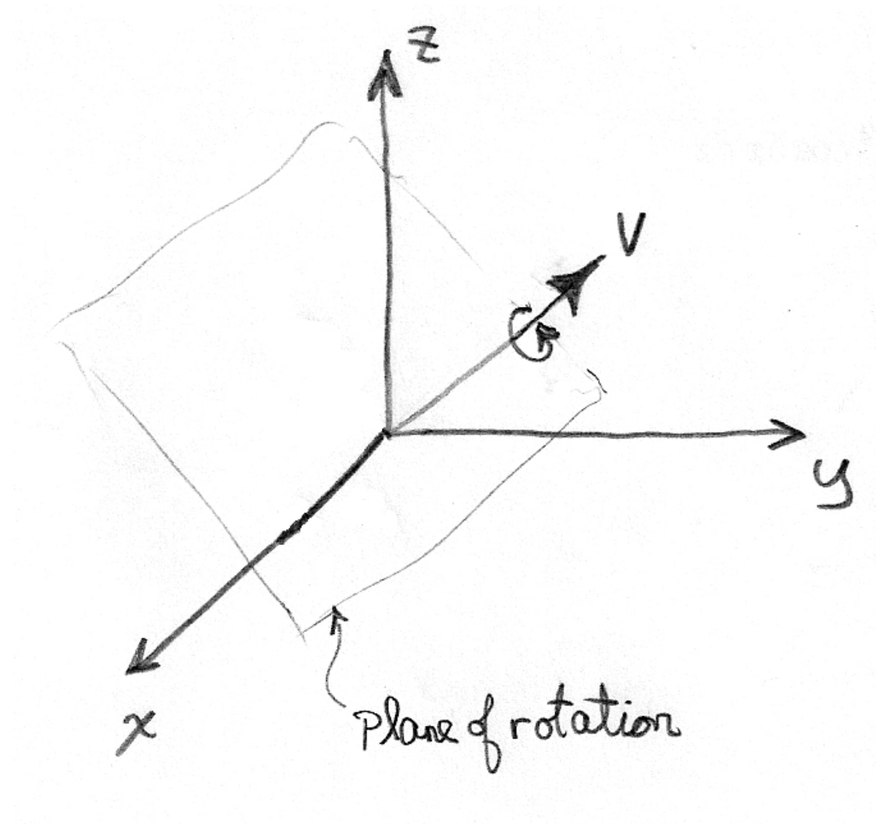
- How well can an irrational number be approximated by rational numbers?

(there are different ‘types’ of irrational numbers)

How ‘close’ to an irrational number can you get using only rational numbers whose denominators are no larger than b? (a/b - type rational numbers)

Some important questions in modern mathematics

- Can every rotation be obtained by rotating around (only) the x, y and z axes?



Some important questions in modern mathematics

- Is the solar system stable? Will the planets continue to orbit the sun in regular patterns forever or will they someday collide?

Some questions in industry where mathematics is used

Some questions in industry where mathematics is used

- Vehicle emission (pollution control)
- How to allocate Intensive Care beds at a hospital to minimize patient waiting times?
- How effective are carbon trading schemes in reducing greenhouse gasses?
- Deciding the best (government) policy for encouraging solar power development
- Why are the tides at the Bay of Fundy so large? (16m) *resonance…..*

Areas of modern mathematics

- Algebra
- Analysis (aka calculus)
- Topology
- Mathematical Logic
- Numerical analysis (using computers)
- Discrete mathematics

http://www.sfu.ca/~rpyke/presentations.html