U3CA22MT

DATA STRUCTURES AND COMPUTER ALGORITHMS

UNIT: I

Definition of a Data structure:

A **data structure** is a specialized format for organizing and storing **data**. General **data structure** types include the array, the file, the record, the table, the tree, and so on. Any **data structure** is designed to organize **data** to suit a specific purpose so that it can be accessed and worked with in appropriate ways.

Data Structure is a way of collecting and organizing data in such a way that we can perform operations on these data in an effective way. Data Structures is about rendering data elements in terms of some relationship, for better organization and storage.

ADT (Abstract data type):

Abstract data type are like user defined data type on which we can perform functions without knowing what is there inside the data type and how the operations are performed on them . As the information is not exposed it's abstracted.

Stack and queue are not a data type they are way of implementing a data structure , but they are called as abstract DATA TYPE because we can do operations on datatype only not on function , here in stack and queue we can do operation like pop , push , enqueue etc. and its ABSTRACT because we are not given information about what is there inside the stack or queue , here we call function for doing operations without knowing what stuffs there in the stack or queue , functions will know what is inside and do the job when called. as all the information is not given only required info is given so its abstract.

Now we'll define three ADTs namely List ADT, Stack ADT, Queue ADT.

List ADT

A list contains elements of same type arranged in sequential order and following operations can be performed on the list.

get() – Return an element from the list at any given position.
insert() – Insert an element at any position of the list.
remove() – Remove the first occurrence of any element from a non-empty list.
removeAt() – Remove the element at a specified location from a non-empty list.
replace() – Replace an element at any position by another element.
size() – Return the number of elements in the list.
isEmpty() – Return true if the list is empty, otherwise return false.
isFull() – Return true if the list is full, otherwise return false.

Stack ADT

A Stack contains elements of same type arranged in sequential order. All operations takes place at a single end that is top of the stack and following operations can be performed:

push() – Insert an element at one end of the stack called top.
pop() – Remove and return the element at the top of the stack, if it is not empty.
peek() – Return the element at the top of the stack without removing it, if the stack is not empty.
size() – Return the number of elements in the stack.
isEmpty() – Return true if the stack is empty, otherwise return false.
isFull() – Return true if the stack is full, otherwise return false.

Queue ADT

A Queue contains elements of same type arranged in sequential order. Operations takes place at both ends, insertion is done at end and deletion is done at front. Following operations can be performed:

enqueue() – Insert an element at the end of the queue.
dequeue() – Remove and return the first element of queue, if the queue is not empty.
peek() – Return the element of the queue without removing it, if the queue is not empty.
size() – Return the number of elements in the queue.
isEmpty() – Return true if the queue is empty, otherwise return false.
isFull() – Return true if the queue is full, otherwise return false.

From these definitions, we can clearly see that the definitions do not specify how these ADTs will be represented and how the operations will be carried out. There can be different ways to implement an ADT, for example, the List ADT can be implemented using arrays, or singly linked list or doubly linked list. Similarly, stack ADT and Queue ADT can be implemented using arrays or linked lists.

Primitive and Composite data types:

Primitive data type is either of the following: a basic **type** is a **data type** provided by a programming language as a basic building block. Most languages allow more complicated **composite types** to be recursively constructed starting from basic **types**.

A **composite data type** or **compound data type** is any data type which can be constructed in a program using the programming language's primitive data types and other composite types. It is sometimes called a **structure** or **aggregate data type**, although the latter term may also refer to arrays, lists, etc.

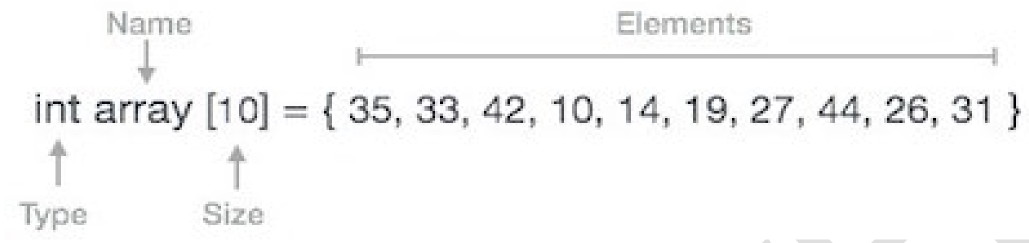
Arrars:

Array is a container which can hold a fix number of items and these items should be of the same type. Most of the data structures make use of arrays to implement their algorithms. Following are the important terms to understand the concept of Array.

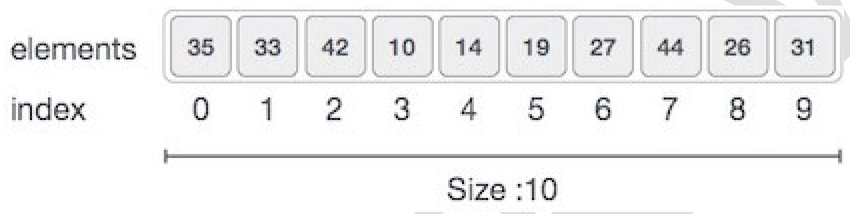
- **Element** − Each item stored in an array is called an element.
- **Index** − Each location of an element in an array has a numerical index, which is used to identify the element.

Array Representation:

Arrays can be declared in various ways in different languages. For illustration, let's take C array declaration.



Arrays can be declared in various ways in different languages. For illustration, let's take C array declaration.



As per the above illustration, following are the important points to be considered.

- Index starts with 0.
- Array length is 10 which means it can store 10 elements.
- Each element can be accessed via its index. For example, we can fetch an element at index 6 as 9.

Basic Operations:

Following are the basic operations supported by an array.

- **Traverse** − print all the array elements one by one.
- **Insertion** − Adds an element at the given index.
- **Deletion** − Deletes an element at the given index.
- **Search** − Searches an element using the given index or by the value.
- **Update** − Updates an element at the given index.

In C, when an array is initialized with size, then it assigns defaults values to its elements in following order.

Data Type	Default Value
bool	false
char	0
int	0
float	0.0
double	0.0f
void	

Insertion Operation

Insert operation is to insert one or more data elements into an array. Based on the requirement, a new element can be added at the beginning, end, or any given index of array.

Example

Algorithm

Let **Array** be a linear unordered array of **MAX** elements.

Let **LA** be a Linear Array (unordered) with **N** elements and **K** is a positive integer such that **K<=N**. Following is the algorithm where ITEM is inserted into the K th position of LA –

```
1. Start
2. Set J = N
3. Set N = N+1
4. Repeat steps 5 and 6 while J >= K
5. Set LA[J+1] = LA[J]
6. Set J = J-1
7. Set LA[K] = ITEM
8. Stop
```

Deletion Operation

Deletion refers to removing an existing element from the array and re-organizing all elements of an array.

Algorithm

Consider **LA** is a linear array with **N** elements and **K** is a positive integer such that **K<=N**. Following is the algorithm to delete an element available at the Kth position of LA.

```
1. Start
2. Set J = K
3. Repeat steps 4 and 5 while J < N
4. Set LA[J] = LA[J + 1]
5. Set J = J+1
6. Set N = N-1
7. Stop
```

Search Operation

You can perform a search for an array element based on its value or its index.

Algorithm

Consider **LA** is a linear array with **N** elements and **K** is a positive integer such that **K<=N**. Following is the algorithm to find an element with a value of ITEM using sequential search.

```
1. Start
2. Set J = 0
3. Repeat steps 4 and 5 while J < N
4. IF LA[J] is equal ITEM THEN GOTO STEP 6
5. Set J = J +1
6. PRINT J, ITEM
7. Stop
```

Update Operation

Update operation refers to updating an existing element from the array at a given index.

Algorithm

Consider **LA** is a linear array with **N** elements and **K** is a positive integer such that **K<=N**. Following is the algorithm to update an element available at the Kth position of LA.

```
1. Start
2. Set LA[K-1] = ITEM
3. Stop
```

Ordered List:

The structure of an ordered list is a collection of items where each item holds a relative position that is based upon some underlying characteristic of the item. The ordering is typically either ascending or descending and we assume that list items have a meaningful comparison operation that is already defined. Many of the ordered list operations are the same as those of the unordered list.

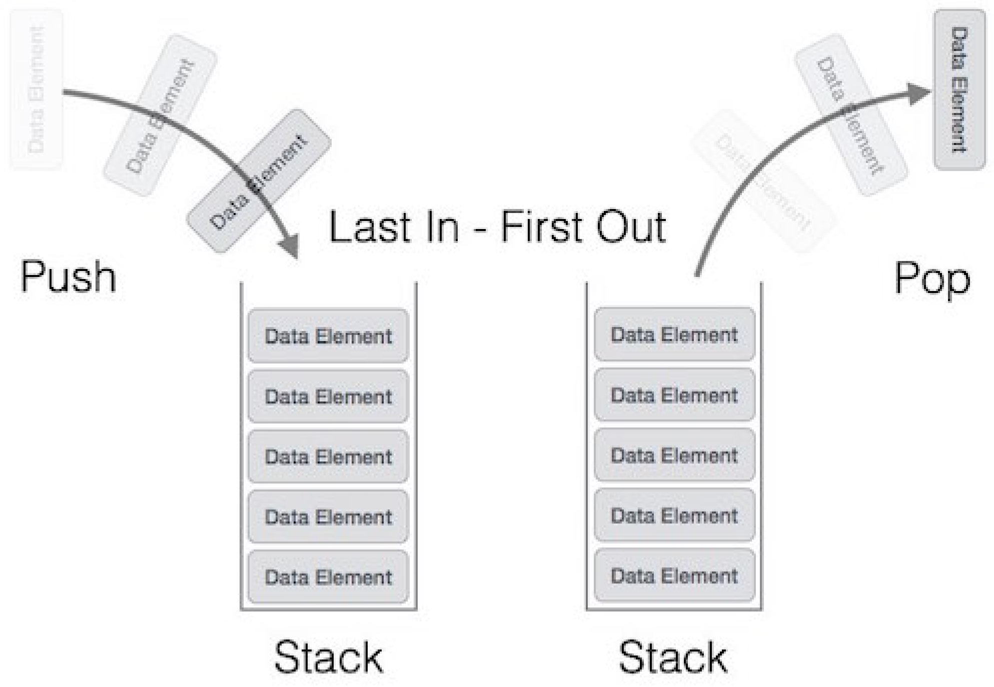
UNIT: II

Stack ADT:

A stack is an Abstract Data Type (ADT), commonly used in most programming languages. It is named stack as it behaves like a real-world stack, for example – a deck of cards or a pile of plates, etc.

A real-world stack allows operations at one end only. For example, we can place or remove a card or plate from the top of the stack only. Likewise, Stack ADT allows all data operations at one end only. At any given time, we can only access the top element of a stack.

This feature makes it LIFO data structure. LIFO stands for Last-in-first-out. Here, the element which is placed (inserted or added) last, is accessed first. In stack terminology, insertion operation is called **PUSH** operation and removal operation is called **POP** operation.



we are going to implement stack using arrays, which makes it a fixed size stack implementation.

Basic Operations

Stack operations may involve initializing the stack, using it and then de-initializing it. Apart from these basic stuffs, a stack is used for the following two primary operations −

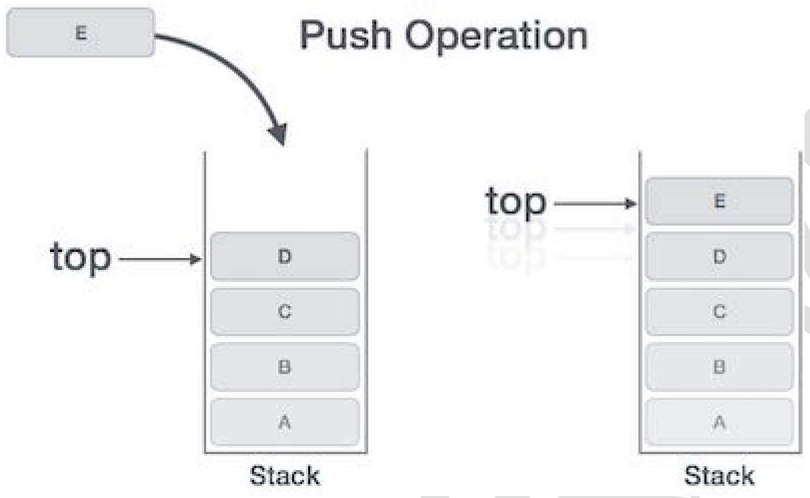
- **push()** − Pushing (storing) an element on the stack.
- **pop()** − Removing (accessing) an element from the stack.

When data is PUSHed onto stack.

Push Operation

The process of putting a new data element onto stack is known as a Push Operation. Push operation involves a series of steps −

Step 1 − Checks if the stack is full.
Step 2 − If the stack is full, produces an error and exit.
Step 3 − If the stack is not full, increments **top** to point next empty space.
Step 4 − Adds data element to the stack location, where top is pointing.
Step 5 − Returns success.



Algorithm for PUSH Operation

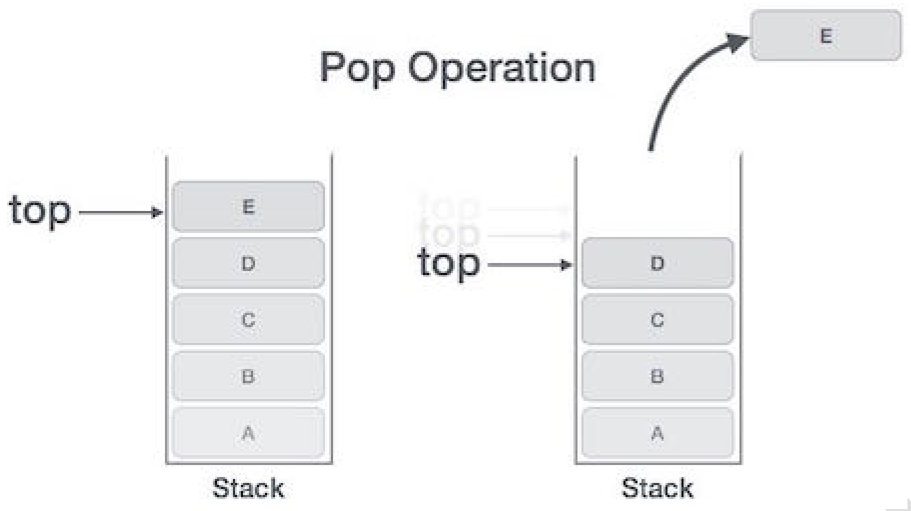
```
begin procedure push: stack, data
   if stack is full
      return null
   endif
   top ← top + 1
   stack[top] ← data
end procedure
```

Pop Operation

Accessing the content while removing it from the stack, is known as a Pop Operation. In an array implementation of pop() operation, the data element is not actually removed, instead **top** is decremented to a lower position in the stack to point to the next value. But in linked-list implementation, pop() actually removes data element and deallocates memory space.

A Pop operation may involve the following steps −

Step 1 − Checks if the stack is empty.
Step 2 − If the stack is empty, produces an error and exit.
Step 3 − If the stack is not empty, accesses the data element at which **top** is pointing.
Step 4 − Decreases the value of top by 1.
Step 5 − Returns success.



Algorithm for Pop Operation

```
begin procedure pop: stack
  if stack is empty
    return null
  endif
    data ← stack[top]
  top ← top - 1
  return data
end procedure
```

Infix to Postfix Conversion:

Any expression can be represented using three types of expressions (Infix, Postfix, and Prefix). We can also convert one type of expression to another type of expression like Infix to Postfix, Infix to Prefix, Postfix to Prefix and vice versa.

To convert any Infix expression into Postfix or Prefix expression we can use the following procedure...
Find all the operators in the given Infix Expression.
Find the order of operators evaluated according to their Operator precedence.
Convert each operator into required type of expression (Postfix or Prefix) in the same order.

Example
Consider the following Infix Expression to be converted into Postfix Expression...

Infix expression	Postfix expression
D = A + B * C	D A B C * + =

Infix to Postfix Conversion using Stack Data Structure

To convert Infix Expression into Postfix Expression using a stack data structure, We can use the following steps...

1. Scan the infix expression from left to right.
2. If the scanned character is an operand, output it.
3. Else,
…..**3.1** If the precedence of the scanned operator is greater than the precedence of the operator in the stack(or the stack is empty or the stack contains a '('), push it.
…..**3.2** Else, Pop all the operators from the stack which are greater than or equal to in precedence than that of the scanned operator. After doing that Push the scanned operator to the stack. (If you encounter parenthesis while popping then stop there and push the scanned operator in the stack.)
4. If the scanned character is an '(', push it to the stack.
5. If the scanned character is an ')', pop the stack and and output it until a '(' is encountered, and discard both the parenthesis.
6. Repeat steps 2-6 until infix expression is scanned.
7. Print the output
8. Pop and output from the stack until it is not empty.

QUEUE ADT
Queue is an abstract data structure, somewhat similar to Stacks. Unlike stacks, a queue is open at both its ends. One end is always used to insert data (enqueue) and the other is used to remove data (dequeue). Queue follows First-In-First-Out methodology, i.e., the data item stored first will be accessed first.
As in stacks, a queue can also be implemented using Arrays, Linked-lists, Pointers and Structures. For the sake of simplicity, we shall implement queues using one-dimensional array.

Basic Operations

Queue operations may involve initializing or defining the queue, utilizing it, and then completely erasing it from the memory. Here we shall try to understand the basic operations associated with queues −

- **enqueue()** − add (store) an item to the queue.
- **dequeue()** − remove (access) an item from the queue.

Few more functions are required to make the above-mentioned queue operation efficient.

These are −

- **peek()** − Gets the element at the front of the queue without removing it.
- **isfull()** − Checks if the queue is full.
- **isempty()** − Checks if the queue is empty.

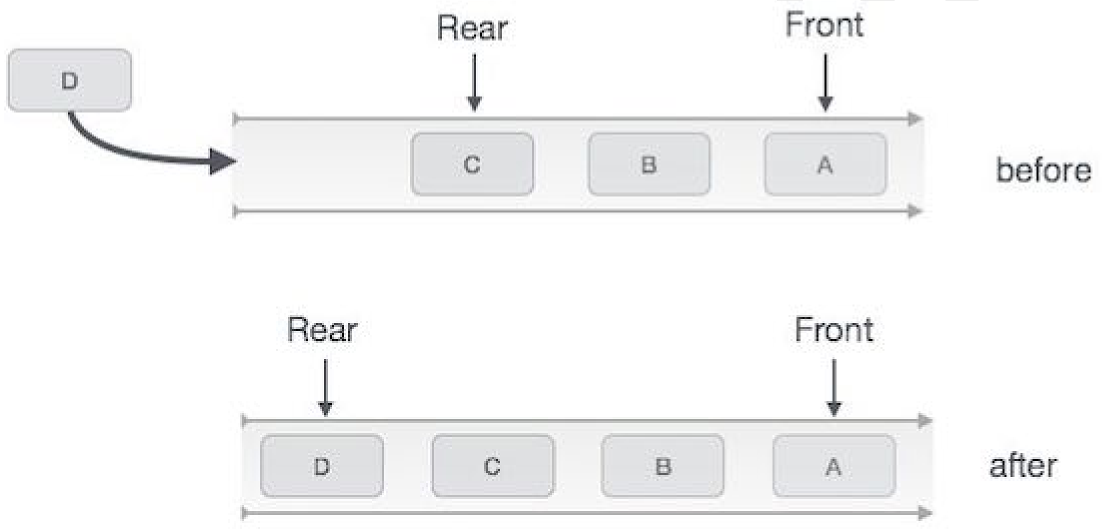
In queue, we always dequeue (or access) data, pointed by **front** pointer and while enqueing (or storing) data in the queue we take help of **rear** pointer.

Enqueue Operation

Queues maintain two data pointers, **front** and **rear**. Therefore, its operations are comparatively difficult to implement than that of stacks.

The following steps should be taken to enqueue (insert) data into a queue −

- **Step 1** − Check if the queue is full.
- **Step 2** − If the queue is full, produce overflow error and exit.
- **Step 3** − If the queue is not full, increment **rear** pointer to point the next empty space.
- **Step 4** − Add data element to the queue location, where the rear is pointing.
- **Step 5** − return success.



Queue Enqueue

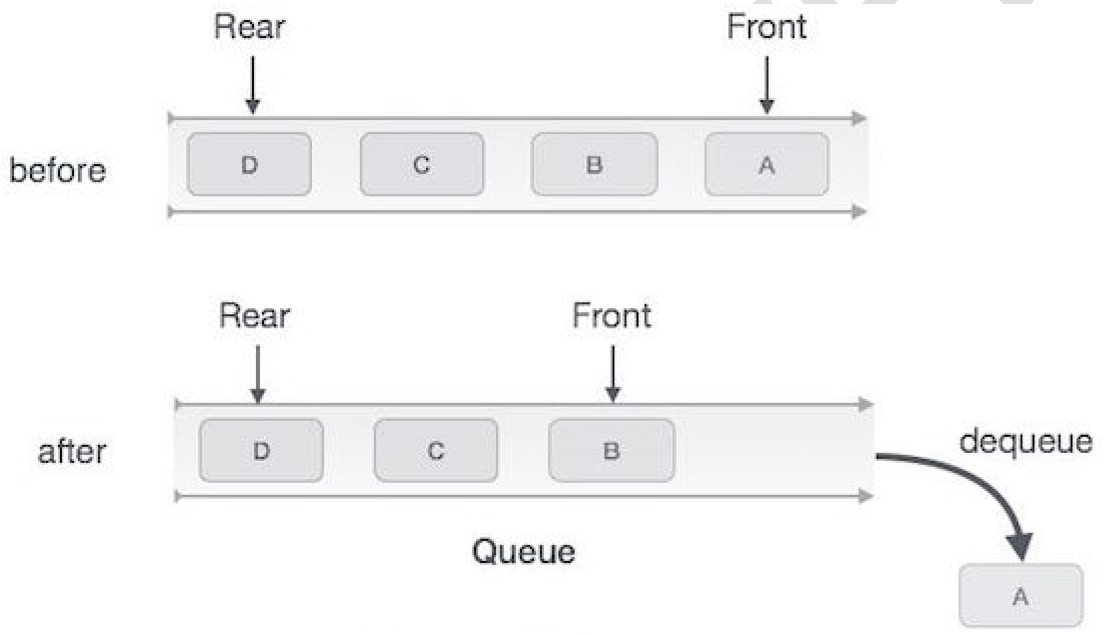
Algorithm for enqueue operation

```
procedure enqueue(data)
      if queue is full
      return overflow
   endif

   rear ← rear + 1
   queue[rear] ← data
   return true
end procedure
```

Dequeue Operation

Accessing data from the queue is a process of two tasks − access the data where **front** is pointing and remove the data after access. The following steps are taken to perform **dequeue** operation −

- **Step 1** − Check if the queue is empty.
- **Step 2** − If the queue is empty, produce underflow error and exit.
- **Step 3** − If the queue is not empty, access the data where **front** is pointing.
- **Step 4** − Increment **front** pointer to point to the next available data element.
- **Step 5** − Return success.



Queue Dequeue

Algorithm for dequeue operation

```
procedure dequeue
     if queue is empty
      return underflow
   end if

   data = queue[front]
   front ← front + 1
   return true

end procedure
```

TYPES OF QUEUE

There are three types of queue:

- Circular Queue
- Priority Queue
- Deque

1. Simple Queue

Simple queue defines the simple operation of queue in which insertion occurs at the rear of the list and deletion occurs at the front of the list.

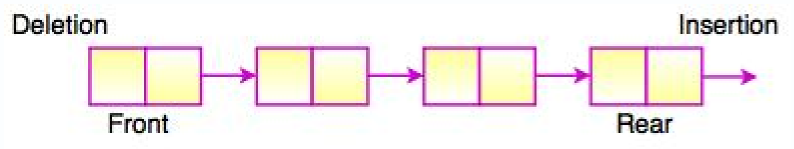


Fig. Simple Queue

2. Circular Queue

A circular queue is one in which the insertion of a element is done at the very first location if the queue if the last location of the queue is full. New element can be inserted if and only if the those location are empty For example, if we have a queue Q of say n elements , then after inserting an element at last location (i.e in the n-1th) location of the array the next element will be inserted at the very first location(i.e at 0^{th} location) of the array.

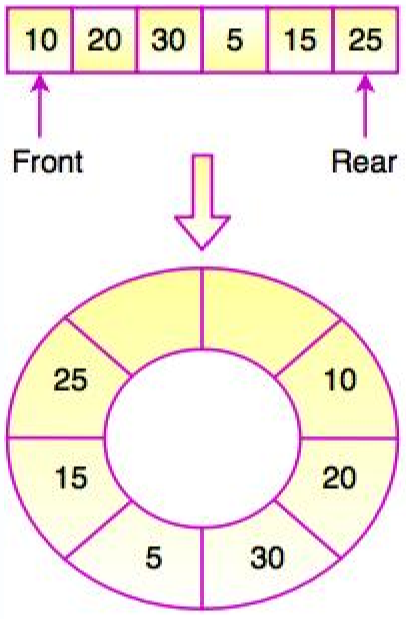


Fig. Circular Queue

A circular queue also have a front and rear to keep the track of the elements to be deleted and inserted and therefore to maintain the unique characteristic of the queue. Following are the assumption made:

- Front will always be pointing to the first element
- If front=rear the queue will be empty
- Each time a new element is inserted into the queue the rear is incremented by one rear=rear+1
- Each time a new element is deleted from the queue the value of front is incremented by one, front=front+1

3. Priority Queue

Priority queue is a collection of elements such that each element has been assigned a priority and the order in which elements are deleted and processed comes from following rules:

- An element of higher priority is processed before any elements of lower priority
- Two elements with the same priority are processed according to the order in which they were added to the queue.

An example of priority queue in computer science occurs in timesharing system in which the processes of higher priority is executed before any process of lower priority. There are two types of priority queue:

- **Ascending priority queue :** It is a collection of items in to which items can be inserted arbitrarily and from which only the smallest item can be removed.
- **Descending priority queue :** It is similar but allows deletion of only the largest item.

4. Double Ended Queue i.e Deque

It is also a homogeneous list of elements in which insertion and deletion operations are performed from both the ends. That is, we can insert elements from the rear end or from the front end. Hence, it is called Double-Ended Queue. There are two types of deques. These two types are due to the restrictions put to perform either the insertions or deletions only at one end.

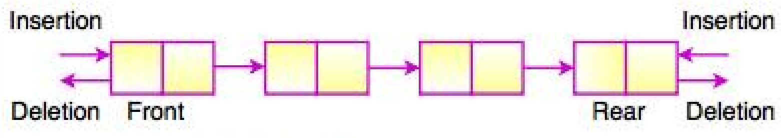


Fig. Double Ended Queue (Dequeue)

- **Input restricted deques :** Input restricted deques allows insertions at only one end of the array or list but deletions allows at both ends.
- **Output restricted deques :** Output restricted deques allows deletions at only one end of the array or list but insertions allow at both ends.

List ADT:

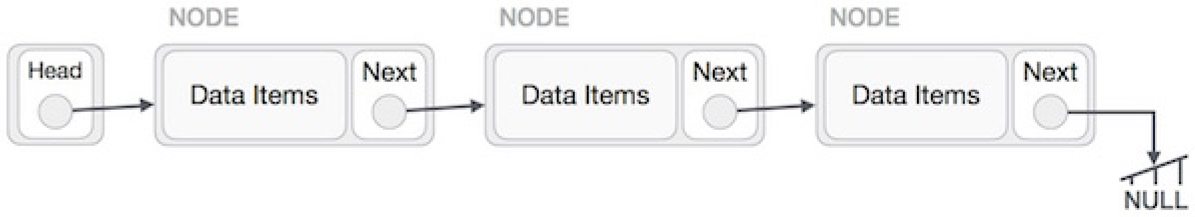
A linked list is a sequence of data structures, which are connected together via links.

Linked List is a sequence of links which contains items. Each link contains a connection to another link. Linked list is the second most-used data structure after array. Following are the important terms to understand the concept of Linked List.

- **Link** − Each link of a linked list can store a data called an element.
- **Next** − Each link of a linked list contains a link to the next link called Next.
- **LinkedList** − A Linked List contains the connection link to the first link called First.

Linked List Representation

Linked list can be visualized as a chain of nodes, where every node points to the next node.



As per the above illustration, following are the important points to be considered.

- Linked List contains a link element called first.
- Each link carries a data field(s) and a link field called next.
- Each link is linked with its next link using its next link.
- Last link carries a link as null to mark the end of the list.

Types of Linked List

Following are the various types of linked list.

- **Simple Linked List** − Item navigation is forward only.
- **Doubly Linked List** − Items can be navigated forward and backward.
- **Circular Linked List** − Last item contains link of the first element as next and the first element has a link to the last element as previous.

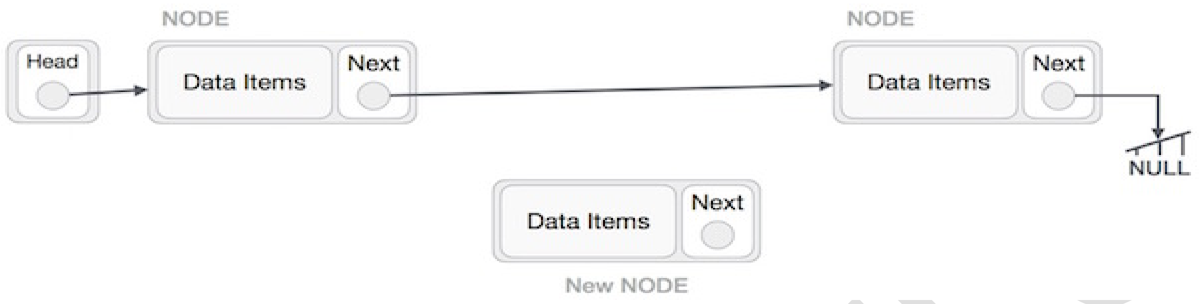
Basic Operations

Following are the basic operations supported by a list.

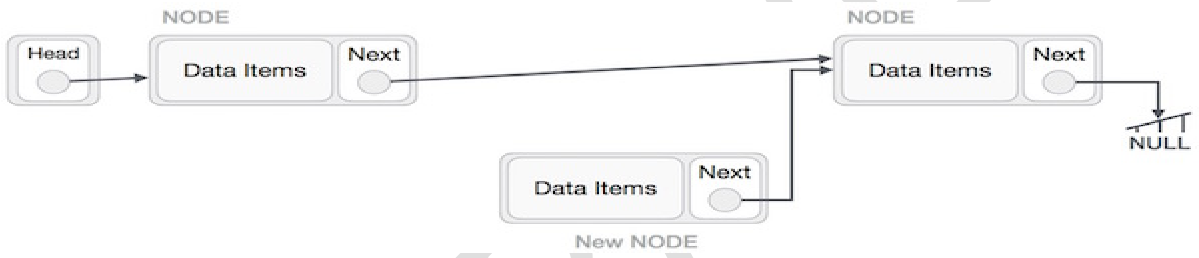
- **Insertion** − Adds an element at the beginning of the list.
- **Deletion** − Deletes an element at the beginning of the list.
- **Display** − Displays the complete list.
- **Search** − Searches an element using the given key.
- **Delete** − Deletes an element using the given key.

Insertion Operation

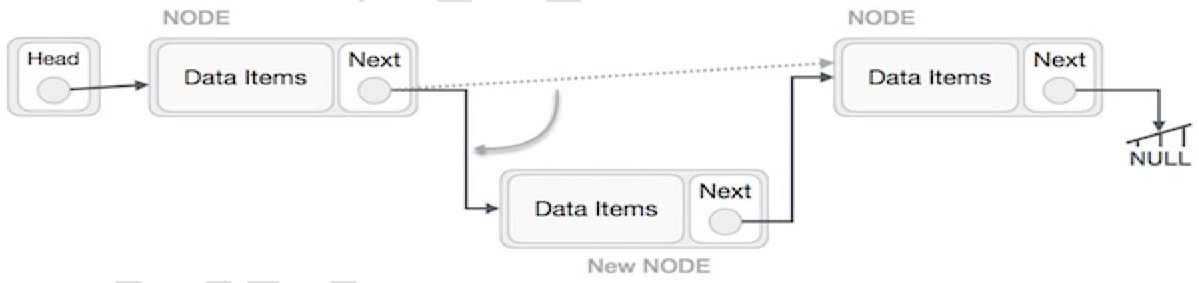
Adding a new node in linked list is a more than one step activity. We shall learn this with diagrams here. First, create a node using the same structure and find the location where it has to be inserted.



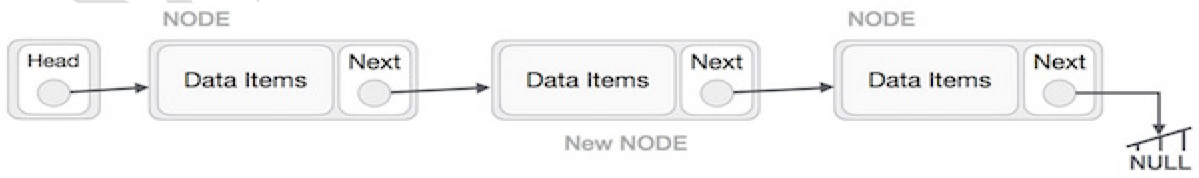
Imagine that we are inserting a node **B** (NewNode), between **A** (LeftNode) and **C** (RightNode). Then point B.next to C –



Now, the next node at the left should point to the new node.



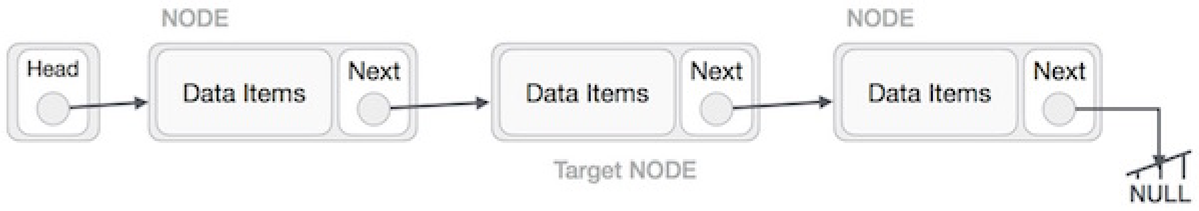
This will put the new node in the middle of the two. The new list should look like this –



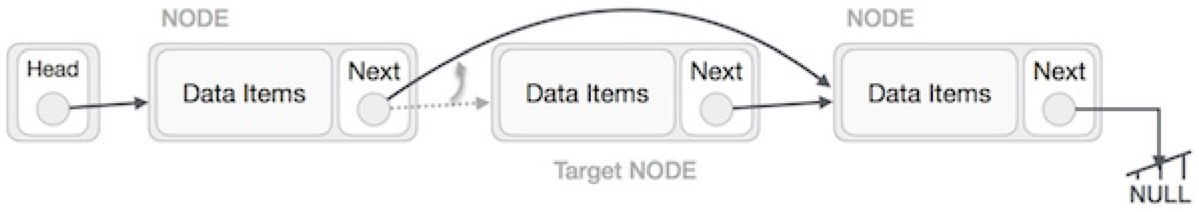
Similar steps should be taken if the node is being inserted at the beginning of the list. While inserting it at the end, the second last node of the list should point to the new node and the new node will point to NULL.

Deletion Operation

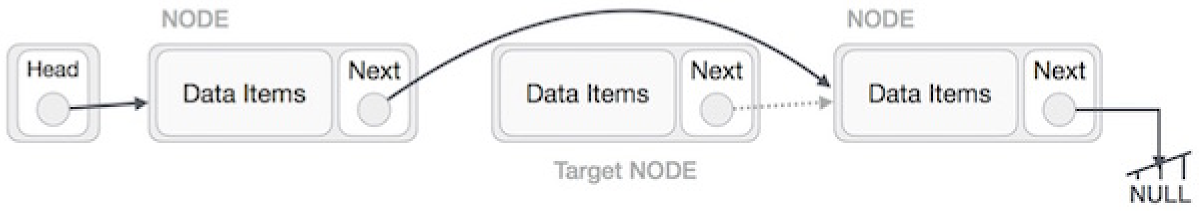
Deletion is also a more than one step process. We shall learn with pictorial representation. First, locate the target node to be removed, by using searching algorithms.



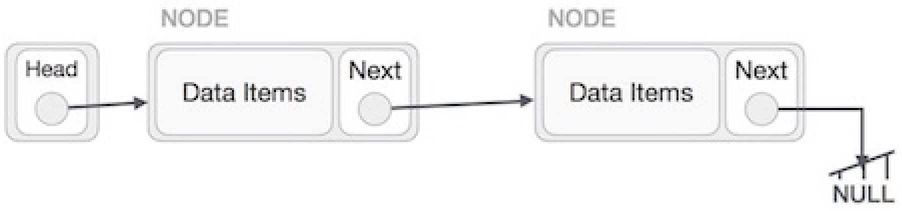
The left (previous) node of the target node now should point to the next node of the target node –



This will remove the link that was pointing to the target node. Now, using the following code, we will remove what the target node is pointing at.



We need to use the deleted node. We can keep that in memory otherwise we can simply deallocate memory and wipe off the target node completely.

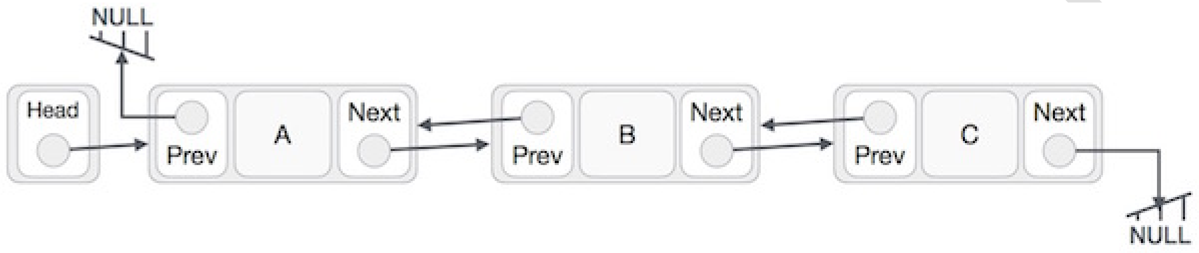


Doubly Linked List

Doubly Linked List is a variation of Linked list in which navigation is possible in both ways, either forward and backward easily as compared to Single Linked List. Following are the important terms to understand the concept of doubly linked list.

- **Link** − Each link of a linked list can store a data called an element.
- **Next** − Each link of a linked list contains a link to the next link called Next.
- **Prev** − Each link of a linked list contains a link to the previous link called Prev.
- **LinkedList** − A Linked List contains the connection link to the first link called First and to the last link called Last.

Doubly Linked List Representation



As per the above illustration, following are the important points to be considered.

- Doubly Linked List contains a link element called first and last.
- Each link carries a data field(s) and two link fields called next and prev.
- Each link is linked with its next link using its next link.
- Each link is linked with its previous link using its previous link.
- The last link carries a link as null to mark the end of the list.

Basic Operations

Following are the basic operations supported by a list.

- **Insertion** − Adds an element at the beginning of the list.
- **Deletion** − Deletes an element at the beginning of the list.
- **Insert Last** − Adds an element at the end of the list.
- **Delete Last** − Deletes an element from the end of the list.
- **Insert After** − Adds an element after an item of the list.
- **Delete** − Deletes an element from the list using the key.
- **Display forward** − Displays the complete list in a forward manner.
- **Display backward** − Displays the complete list in a backward manner.

Insertion Operation

Following code demonstrates the insertion operation at the beginning of a doubly linked list.

Example

```
//insert link at the first location
void insertFirst(int key, int data) {

   //create a link
   struct node *link = (struct node*) malloc(sizeof(struct node));
   link->key = key;
   link->data = data;

   if(isEmpty()) {
      //make it the last link
      last = link;
   } else {
      //update first prev link
      head->prev = link;
   }

   //point it to old first link
   link->next = head;

   //point first to new first link
   head = link;
}
```

Deletion Operation

Following code demonstrates the deletion operation at the beginning of a doubly linked list.

Example

```
//delete first item
struct node* deleteFirst() {

   //save reference to first link
   struct node *tempLink = head;

   //if only one link
   if(head->next == NULL) {
      last = NULL;
   } else {
      head->next->prev = NULL;
   }

   head = head->next;

   //return the deleted link
   return tempLink;
}
```

Insertion at the End of an Operation

Following code demonstrates the insertion operation at the last position of a doubly linked list.

Example

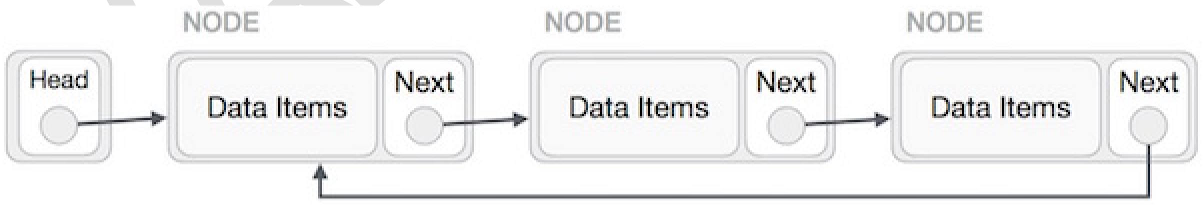
```c
//insert link at the last location
void insertLast(int key, int data) {

   //create a link
   struct node *link = (struct node*) malloc(sizeof(struct node));
   link->key = key;
   link->data = data;

   if(isEmpty()) {
      //make it the last link
      last = link;
   } else {
      //make link a new last link
      last->next = link;

      //mark old last node as prev of new link
      link->prev = last;
   }

   //point last to new last node
   last = link;
}
```

Circular Linked List

Circular Linked List is a variation of Linked list in which the first element points to the last element and the last element points to the first element. Both Singly Linked List and Doubly Linked List can be made into a circular linked list.

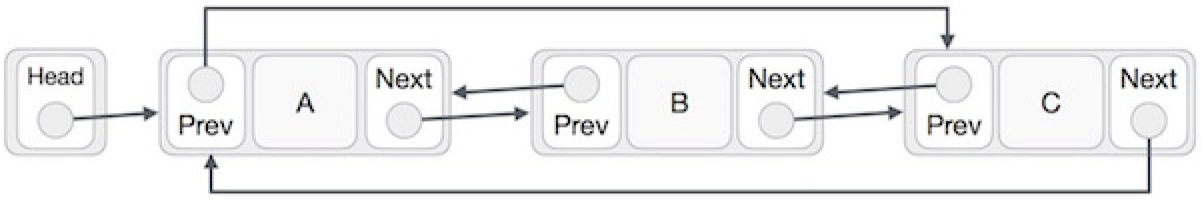
Singly Linked List as Circular

In singly linked list, the next pointer of the last node points to the first node.



Doubly Linked List as Circular

In doubly linked list, the next pointer of the last node points to the first node and the previous pointer of the first node points to the last node making the circular in both directions.



As per the above illustration, following are the important points to be considered.

- The last link's next points to the first link of the list in both cases of singly as well as doubly linked list.
- The first link's previous points to the last of the list in case of doubly linked list.

Basic Operations

Following are the important operations supported by a circular list.

- **insert** − Inserts an element at the start of the list.
- **delete** − Deletes an element from the start of the list.
- **display** − Displays the list.

Insertion Operation

Following code demonstrates the insertion operation in a circular linked list based on single linked list.

Example

```
//insert link at the first location
void insertFirst(int key, int data) {
   //create a link
   struct node *link = (struct node*) malloc(sizeof(struct node));
   link->key = key;
   link->data= data;

   if (isEmpty()) {
      head = link;
      head->next = head;
   } else {
      //point it to old first node
      link->next = head;

      //point first to new first node
      head = link;
   }
}
```

Deletion Operation

Following code demonstrates the deletion operation in a circular linked list based on single linked list.

```
//delete first item
struct node * deleteFirst() {
   //save reference to first link
   struct node *tempLink = head;

   if(head->next == head) {
      head = NULL;
      return tempLink;
   }
   //mark next to first link as first
   head = head->next;

   //return the deleted link
   return tempLink;
}
```

Display List Operation

Following code demonstrates the display list operation in a circular linked list.

```
//display the list
void printList() {
   struct node *ptr = head;
   printf("\n[ ");
   //start from the beginning
   if(head != NULL) {
      while(ptr->next != ptr) {
         printf("(%d,%d) ",ptr->key,ptr->data);
         ptr = ptr->next;
      }
   }
   printf(" ]");
}
```

Adding two polynomials using Linked List

Given two polynomial numbers represented by a linked list. Write a function that add these lists means add the coefficients who have same variable powers.

Example:

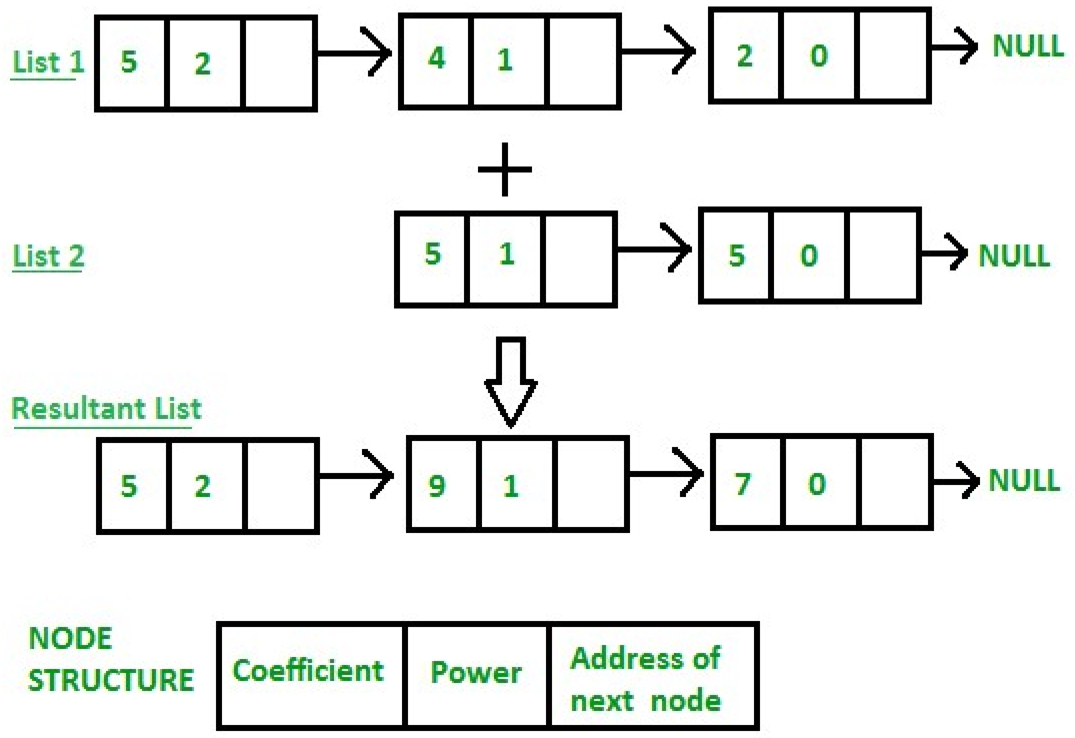
Input:

1st number = 5x^2 + 4x^1 + 2x^0

2nd number = 5x^1 + 5x^0

Output:

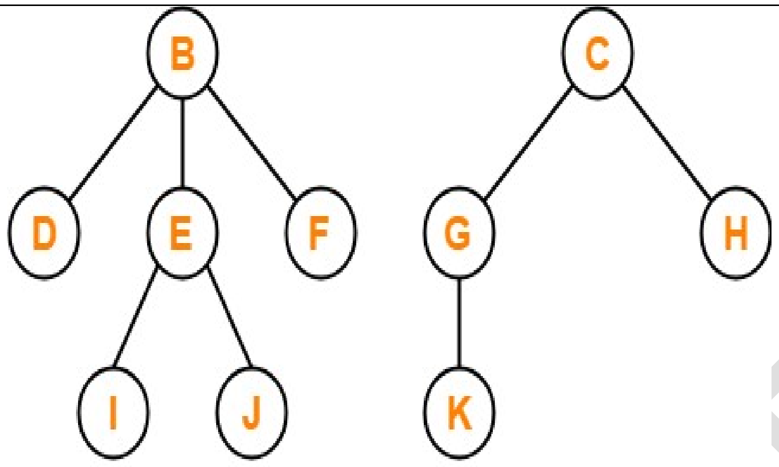
5x^2 + 9x^1 + 7x^0


List 1
5
2
4
1
2
0
NULL
+
List 2
5
1
5
0
NULL
Resultant List
5
2
9
1
7
0
NULL
NODE STRUCTURE
Coefficient
Power
Address of next node

UNIT: III

Trees

Tree terminology

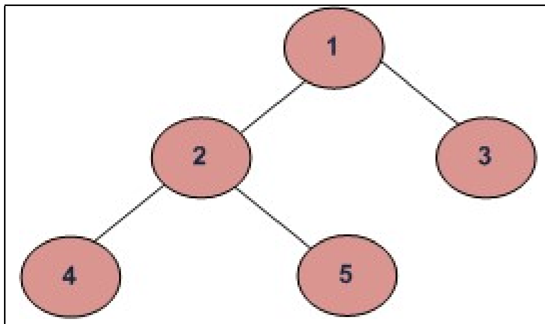
 Forest	**1 Root** **2 Edge** **3 Parent** **4 Child** **5 Siblings** **6 Degree** **7 Internal Node** **8 Leaf Node** **9 Level** **10 Height** **11 Depth** **12 Sub Tree** **13 Forest**

Tree Traversal

Traversal is a process to visit all the nodes of a tree and may print their values too. Because, all nodes are connected via edges (links) we always start from the root (head) node. That is, we cannot randomly access a node in a tree. There are three ways which we use to traverse a tree −

- Pre-order Traversal
- In-order Traversal
- Post-order Traversal

Generally, we traverse a tree to search or locate a given item or key in the tree or to print all the values it contains.

	**Preorder (Root, Left, Right) : 1 2 4 5 3** **Inorder (Left, Root, Right) : 4 2 5 1 3** **Postorder (Left, Right, Root) : 4 5 2 3 1**

Binary Search Trees

A Binary Search Tree (BST) is a tree in which all the nodes follow the below-mentioned properties −

- The left sub-tree of a node has a key less than or equal to its parent node's key.
- The right sub-tree of a node has a key greater than to its parent node's key.

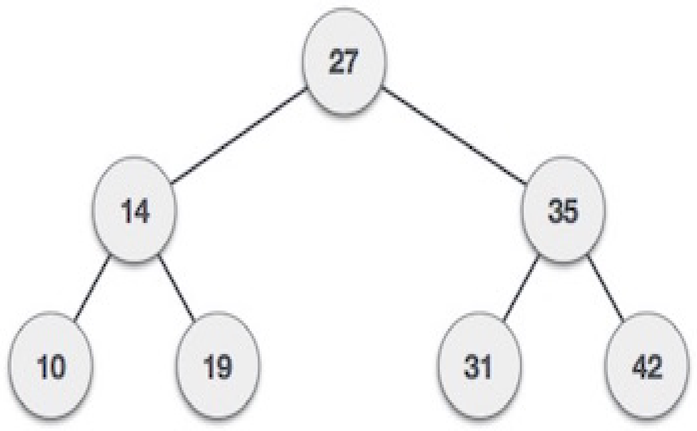
Thus, BST divides all its sub-trees into two segments; the left sub-tree and the right sub-tree and can be defined as −

```
left_subtree (keys) ≤ node (key) ≤ right_subtree (keys)
```

Representation

BST is a collection of nodes arranged in a way where they maintain BST properties. Each node has a key and an associated value. While searching, the desired key is compared to the keys in BST and if found, the associated value is retrieved.

Following is a pictorial representation of BST –



Basic Operations

Following are the basic operations of a tree −

- **Search** − Searches an element in a tree.
- **Insert** − Inserts an element in a tree.
- **Pre-order Traversal** − Traverses a tree in a pre-order manner.
- **In-order Traversal** − Traverses a tree in an in-order manner.
- **Post-order Traversal** − Traverses a tree in a post-order manner.

Node

Define a node having some data, references to its left and right child nodes.

```
struct node
 {
  int data;
  struct node *leftChild;
  struct node *rightChild;
};
```

Search Operation

Whenever an element is to be searched, start searching from the root node. Then if the data is less than the key value, search for the element in the left subtree. Otherwise, search for the element in the right subtree. Follow the same algorithm for each node.

Algorithm

```
struct node* search(int data){
  struct node *current = root;
  printf("Visiting elements: ");
  while(current->data != data){
    if(current != NULL) {
      printf("%d ",current->data);
      //go to left tree
      if(current->data > data){
        current = current->leftChild;
      }  //else go to right tree
      else {
        current = current->rightChild;
      }
      //not found
      if(current == NULL){
        return NULL;
      }
    }
  }

  return current;
}
```

Insert Operation

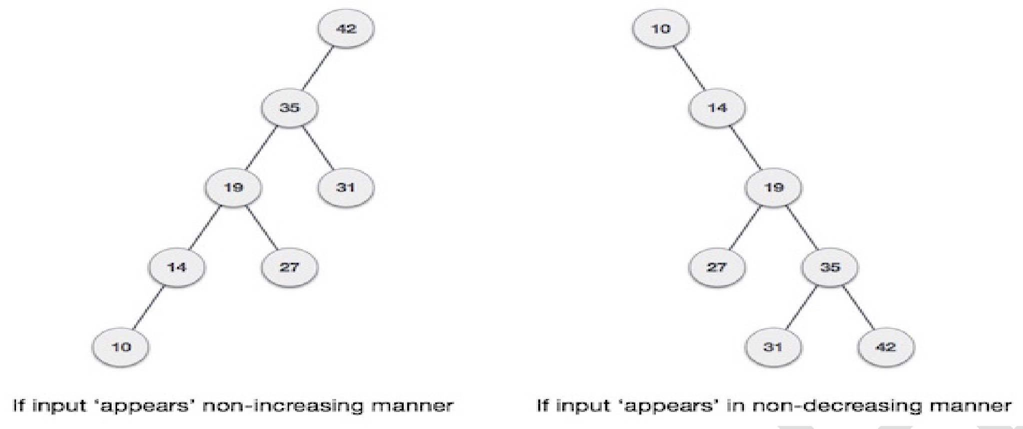
Whenever an element is to be inserted, first locate its proper location. Start searching from the root node, then if the data is less than the key value, search for the empty location in the left subtree and insert the data. Otherwise, search for the empty location in the right subtree and insert the data.

Algorithm

```
void insert(int data) {
   struct node *tempNode = (struct node*) malloc(sizeof(struct node));
   struct node *current;
   struct node *parent;

   tempNode->data = data;
   tempNode->leftChild = NULL;
   tempNode->rightChild = NULL;

   //if tree is empty
   if(root == NULL) {
      root = tempNode;
   } else {
      current = root;
      parent = NULL;

      while(1) {
         parent = current;

         //go to left of the tree
         if(data < parent->data) {
            current = current->leftChild;
            //insert to the left

            if(current == NULL) {
               parent->leftChild = tempNode;
               return;
            }
         }  //go to right of the tree
         else {
            current = current->rightChild;
//insert to the right
            if(current == NULL) {
               parent->rightChild = tempNode;
               return;
            }
         }
      }
   }
}
```

AVL Trees

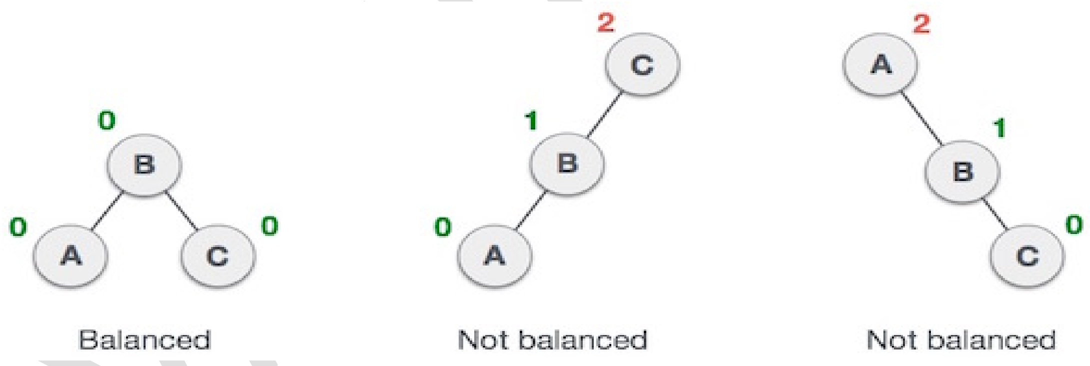
What if the input to binary search tree comes in a sorted (ascending or descending) manner? It will then look like this –


If input 'appears' non-increasing manner
If input 'appears' in non-decreasing manner

It is observed that BST's worst-case performance is closest to linear search algorithms, that is O(n). In real-time data, we cannot predict data pattern and their frequencies. So, a need arises to balance out the existing BST.

Named after their inventor **Adelson**, **Velski** & **Landis**, **AVL trees** are height balancing binary search tree. AVL tree checks the height of the left and the right sub-trees and assures that the difference is not more than 1. This difference is called the **Balance Factor**.

Here we see that the first tree is balanced and the next two trees are not balanced −



In the second tree, the left subtree of **C** has height 2 and the right subtree has height 0, so the difference is 2. In the third tree, the right subtree of **A** has height 2 and the left is missing, so it is 0, and the difference is 2 again. AVL tree permits difference (balance factor) to be only 1.

BalanceFactor = height(left-sutree) − height(right-sutree)

If the difference in the height of left and right sub-trees is more than 1, the tree is balanced using some rotation techniques.

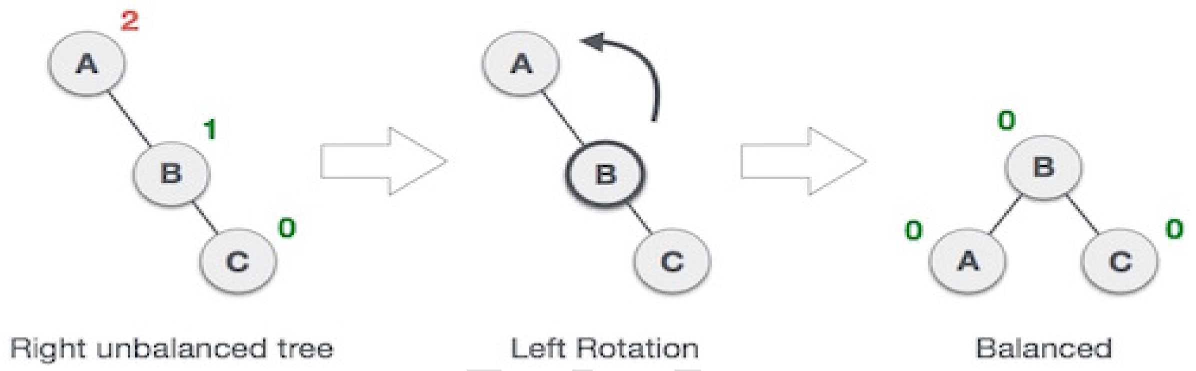
AVL Rotations

To balance itself, an AVL tree may perform the following four kinds of rotations −

- Left rotation
- Right rotation
- Left-Right rotation
- Right-Left rotation
- The first two rotations are single rotations and the next two rotations are double rotations. To have an unbalanced tree, we at least need a tree of height 2. With this simple tree, let's understand them one by one.

Left Rotation

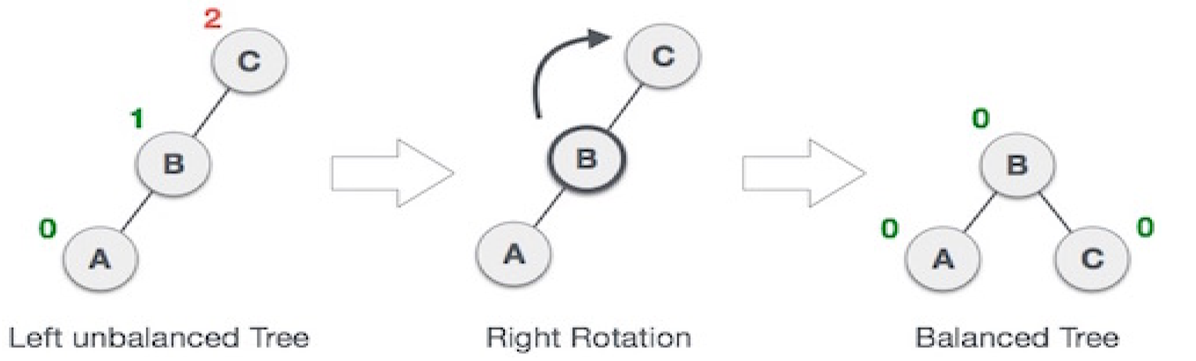
If a tree becomes unbalanced, when a node is inserted into the right subtree of the right subtree, then we perform a single left rotation −



In our example, node **A** has become unbalanced as a node is inserted in the right subtree of A's right subtree. We perform the left rotation by making **A**the left-subtree of B.

Right Rotation

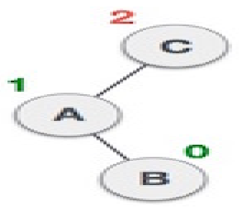
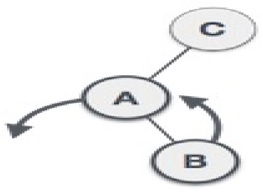
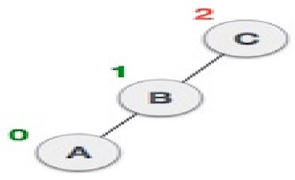
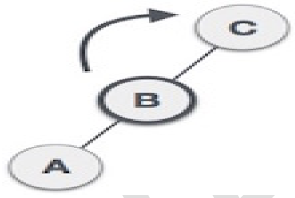
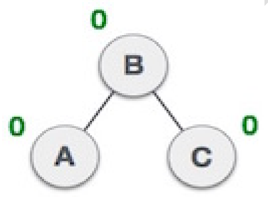
AVL tree may become unbalanced, if a node is inserted in the left subtree of the left subtree. The tree then needs a right rotation.



As depicted, the unbalanced node becomes the right child of its left child by performing a right rotation.

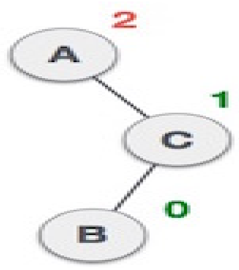
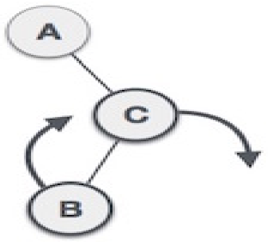
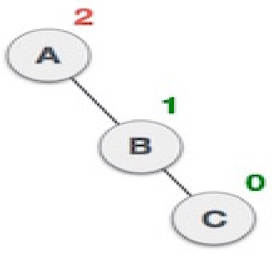
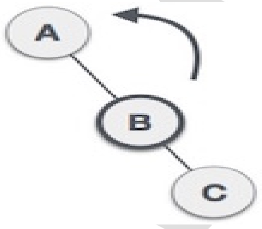
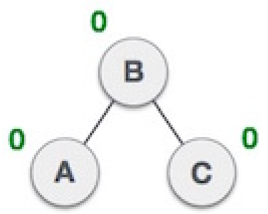
Left-Right Rotation

Double rotations are slightly complex version of already explained versions of rotations. To understand them better, we should take note of each action performed while rotation. Let's first check how to perform Left-Right rotation. A left-right rotation is a combination of left rotation followed by right rotation.

State	Action
	A node has been inserted into the right subtree of the left subtree. This makes **C** an unbalanced node. These scenarios cause AVL tree to perform left-right rotation.
	We first perform the left rotation on the left subtree of **C**. This makes **A**, the left subtree of **B**.
	Node **C** is still unbalanced, however now, it is because of the left-subtree of the left-subtree.
	We shall now right-rotate the tree, making **B** the new root node of this subtree. **C** now becomes the right subtree of its own left subtree.
	The tree is now balanced.

Right-Left Rotation

The second type of double rotation is Right-Left Rotation. It is a combination of right rotation followed by left rotation.

State	Action
	A node has been inserted into the left subtree of the right subtree. This makes **A**, an unbalanced node with balance factor 2.
	First, we perform the right rotation along **C** node, making **C** the right subtree of its own left subtree **B**. Now, **B** becomes the right subtree of **A**.
	Node **A** is still unbalanced because of the right subtree of its right subtree and requires a left rotation.
	A left rotation is performed by making **B** the new root node of the subtree. **A** becomes the left subtree of its right subtree **B**.
	The tree is now balanced.

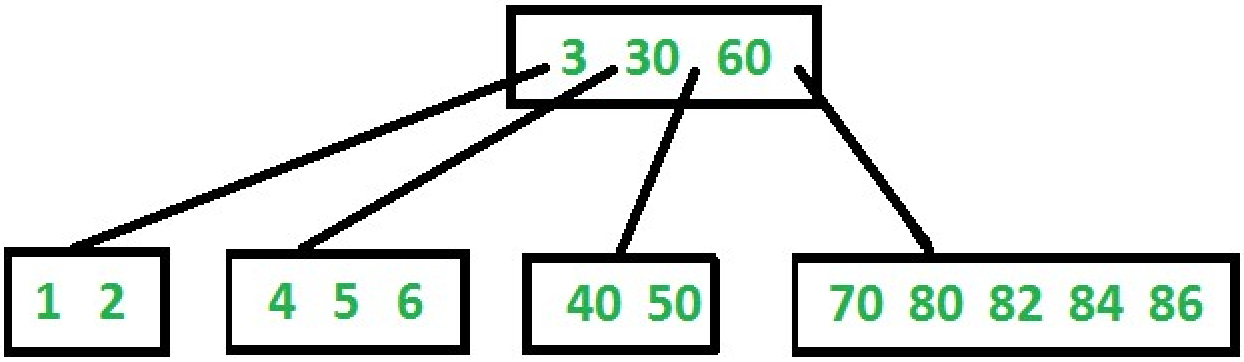
B-Tree

B-Tree is a self-balancing search tree. In most of the other self-balancing search trees (like AVL and Red-Black Trees), it is assumed that everything is in main memory. To understand the use of B-Trees, we must think of the huge amount of data that cannot fit in main memory. When the number of keys is high, the data is read from disk in the form of blocks. Disk access time is very high compared to main memory access time. The main idea of using B-Trees is to reduce the number of disk accesses. Most of the tree operations (search, insert, delete, max, min, ..etc) require O(h) disk accesses where h is the height of the tree. B-tree is a fat tree. The height of B-Trees is kept low by putting maximum possible keys in a B-Tree node. Generally, a B-Tree node size is kept equal to the disk block size. Since h is low for B-Tree, total disk accesses for most of the operations are reduced significantly compared to balanced Binary Search Trees like AVL Tree, Red-Black Tree, ..etc.

Properties of B-Tree

1) All leaves are at same level.
2) A B-Tree is defined by the term *minimum degree* ‘t’. The value of t depends upon disk block size.
3) Every node except root must contain at least t-1 keys. Root may contain minimum 1 key.
4) All nodes (including root) may contain at most 2t – 1 keys.
5) Number of children of a node is equal to the number of keys in it plus 1.
6) All keys of a node are sorted in increasing order. The child between two keys k1 and k2 contains all keys in the range from k1 and k2.
7) B-Tree grows and shrinks from the root which is unlike Binary Search Tree. Binary Search Trees grow downward and also shrink from downward.
8) Like other balanced Binary Search Trees, time complexity to search, insert and delete is O(Logn).

Following is an example B-Tree of minimum degree 3. Note that in practical B-Trees, the value of minimum degree is much more than 3.



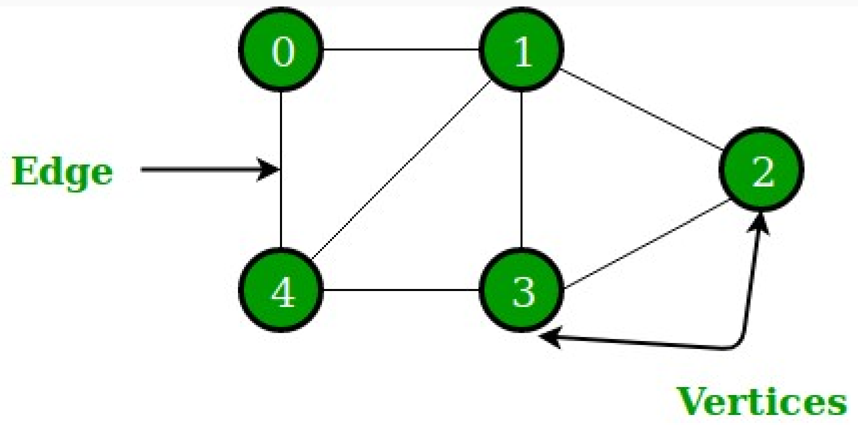
Search
Search is similar to the search in Binary Search Tree. Let the key to be searched be k. We start from the root and recursively traverse down. For every visited non-leaf node, if the node has the key, we simply return the node. Otherwise, we recur down to the appropriate child (The child which is just before the first greater key) of the node. If we reach a leaf node and don't find k in the leaf node, we return NULL.

Traverse
Traversal is also similar to Inorder traversal of Binary Tree. We start from the leftmost child, recursively print the leftmost child, then repeat the same process for remaining children and keys. In the end, recursively print the rightmost child.

Graphs:

A Graph is a non-linear data structure consisting of nodes and edges. The nodes are sometimes also referred to as vertices and the edges are lines or arcs that connect any two nodes in the graph. More formally a Graph can be defined as,

A Graph consists of a finite set of vertices(or nodes) and set of Edges which connect a pair of nodes.



In the above Graph, the set of vertices V = {0,1,2,3,4} and the set of edges E = {01, 12, 23, 34, 04, 14, 13}.

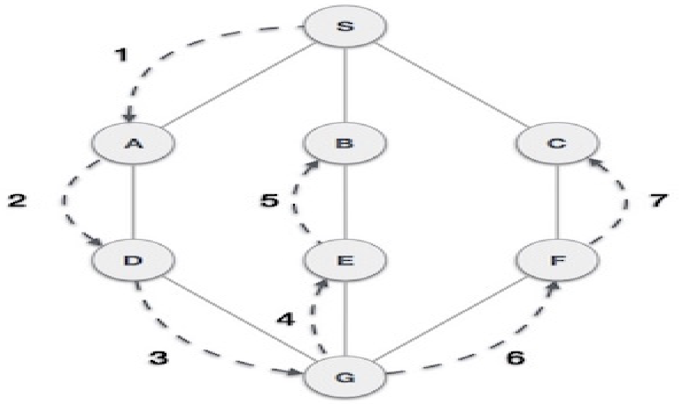
Graphs are used to solve many real-life problems. Graphs are used to represent networks. The networks may include paths in a city or telephone network or circuit network. Graphs are also used in social networks like linkedIn, Facebook. For example, in Facebook, each person is represented with a vertex(or node). Each node is a structure and contains information like person id, name, gender, locale etc.

Undirected graphs have edges that do not have a direction. The edges indicate a *two-way* relationship, in that each edge can be traversed in both directions.

Directed graphs have edges with direction. The edges indicate a *one-way* relationship, in that each edge can only be traversed in a single direction.

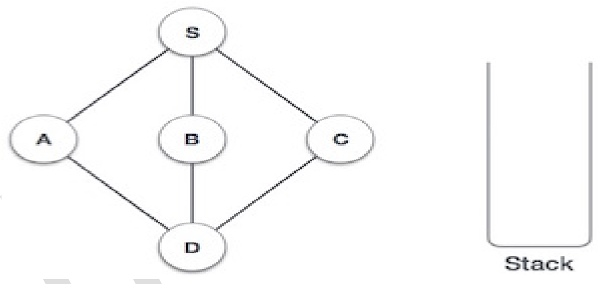
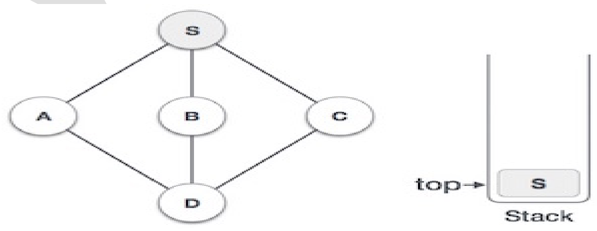
Depth First Search

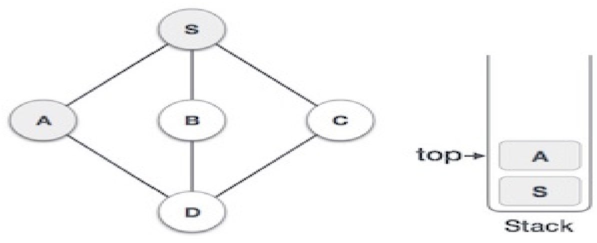
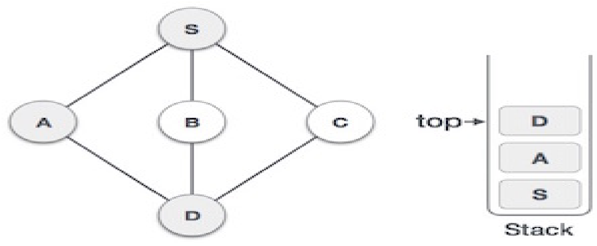
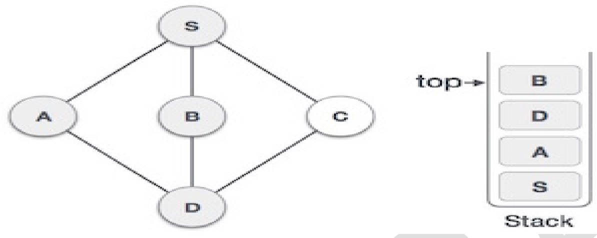
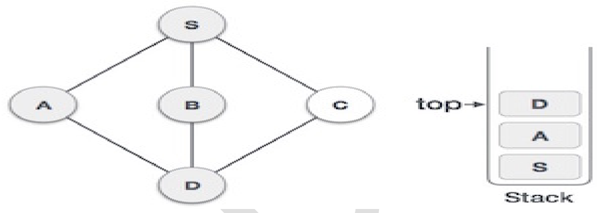
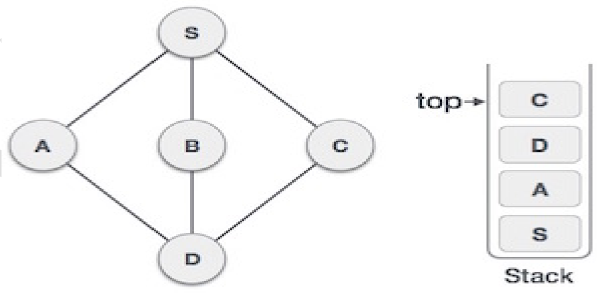
Depth First Search (DFS) algorithm traverses a graph in a depthward motion and uses a stack to remember to get the next vertex to start a search, when a dead end occurs in any iteration.



As in the example given above, DFS algorithm traverses from S to A to D to G to E to B first, then to F and lastly to C. It employs the following rules.

- **Rule 1** − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Push it in a stack.
- **Rule 2** − If no adjacent vertex is found, pop up a vertex from the stack. (It will pop up all the vertices from the stack, which do not have adjacent vertices.)
- **Rule 3** − Repeat Rule 1 and Rule 2 until the stack is empty.

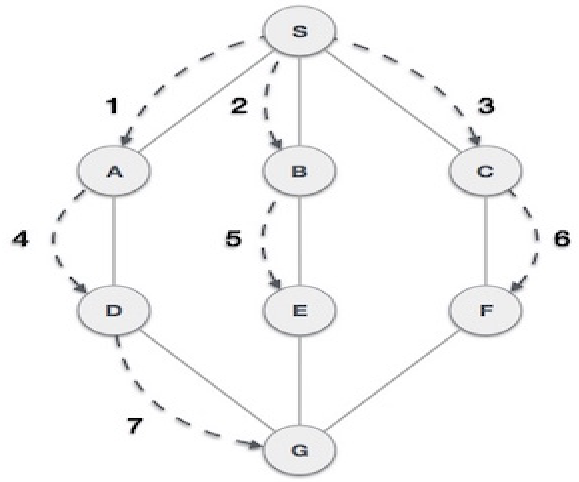
Step	Traversal	Description
1		Initialize the stack.
2		Mark **S** as visited and put it onto the stack. Explore any unvisited adjacent node from **S**. We have three nodes and we can pick any of them. For this example, we shall take the node in an alphabetical order.

3		Mark **A** as visited and put it onto the stack. Explore any unvisited adjacent node from A. Both **S**and **D** are adjacent to **A** but we are concerned for unvisited nodes only.
4		Visit **D** and mark it as visited and put onto the stack. Here, we have **B** and **C** nodes, which are adjacent to **D** and both are unvisited. However, we shall again choose in an alphabetical order.
5		We choose **B**, mark it as visited and put onto the stack. Here **B**does not have any unvisited adjacent node. So, we pop **B**from the stack.
6		We check the stack top for return to the previous node and check if it has any unvisited nodes. Here, we find **D** to be on the top of the stack.
7		Only unvisited adjacent node is from **D** is **C** now. So we visit **C**, mark it as visited and put it onto the stack.

As **C** does not have any unvisited adjacent node so we keep popping the stack until we find a node that has an unvisited adjacent node. In this case, there's none and we keep popping until the stack is empty.

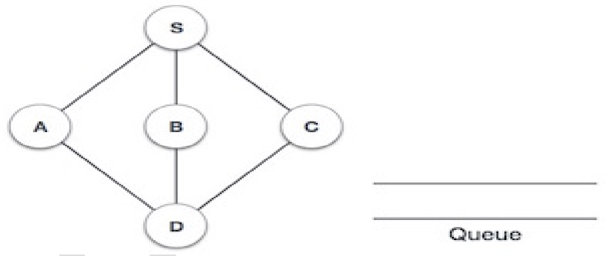
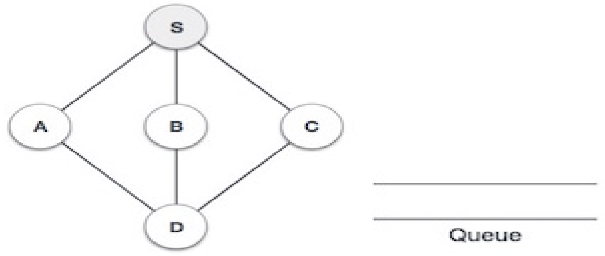
Breadth First Search

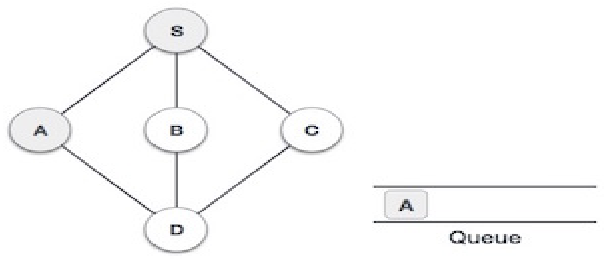
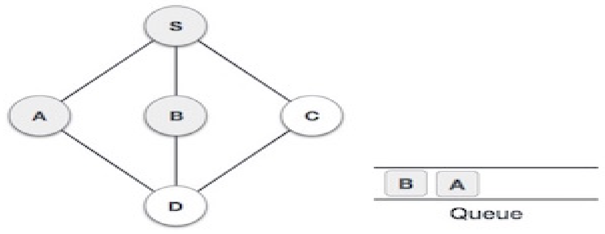
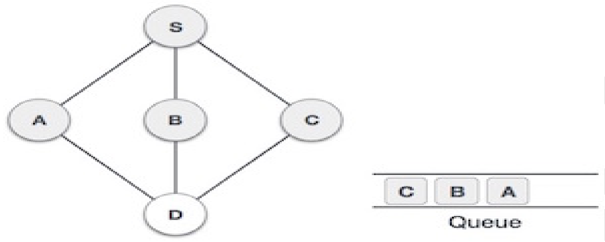
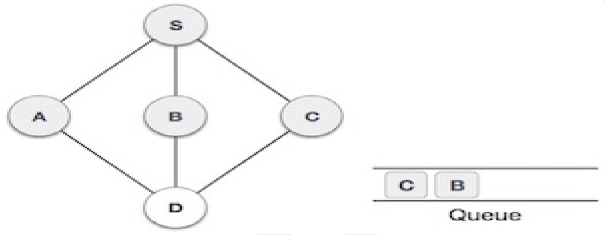
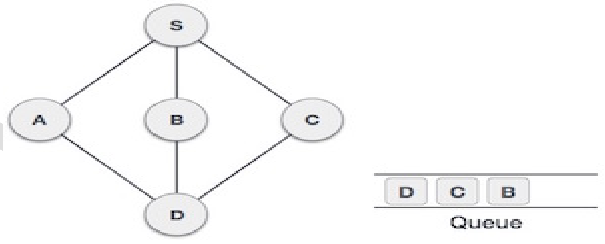
Breadth First Search (BFS) algorithm traverses a graph in a breadthward motion and uses a queue to remember to get the next vertex to start a search, when a dead end occurs in any iteration.



As in the example given above, BFS algorithm traverses from A to B to E to F first then to C and G lastly to D. It employs the following rules.

- **Rule 1** − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Insert it in a queue.
- **Rule 2** − If no adjacent vertex is found, remove the first vertex from the queue.
- **Rule 3** − Repeat Rule 1 and Rule 2 until the queue is empty.

Step	**Traversal**	**Description**
1		Initialize the queue.
2		We start from visiting **S**(starting node), and mark it as visited.

3		We then see an unvisited adjacent node from **S**. In this example, we have three nodes but alphabetically we choose **A**, mark it as visited and enqueue it.
4		Next, the unvisited adjacent node from **S** is **B**. We mark it as visited and enqueue it.
5		Next, the unvisited adjacent node from **S** is **C**. We mark it as visited and enqueue it.
6		Now, **S** is left with no unvisited adjacent nodes. So, we dequeue and find **A**.
7		From **A** we have **D** as unvisited adjacent node. We mark it as visited and enqueue it.

At this stage, we are left with no unmarked (unvisited) nodes. But as per the algorithm we keep on dequeuing in order to get all unvisited nodes. When the queue gets emptied, the program is over.

Dijkstra's shortest path algorithm

Given a graph and a source vertex in the graph, find shortest paths from source to all vertices in the given graph.

Dijkstra's algorithm, We maintain two sets, one set contains vertices included in shortest path tree, other set includes vertices not yet included in shortest path tree. At every step of the algorithm, we find a vertex which is in the other set (set of not yet included) and has a minimum distance from the source.

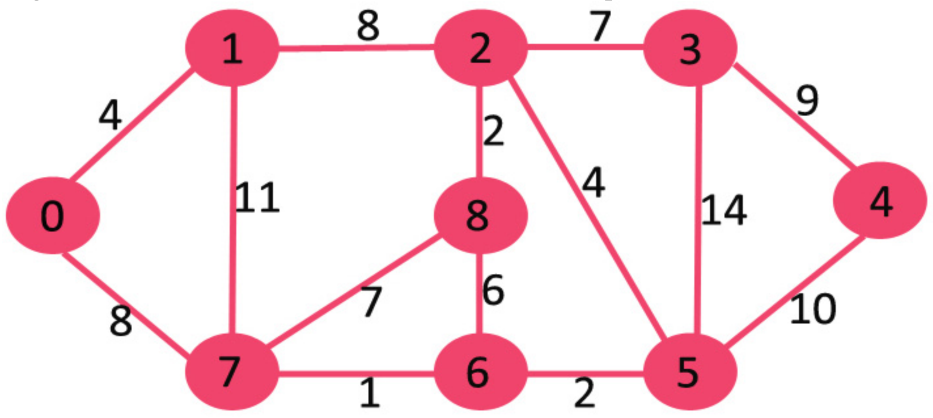
Below are the detailed steps used in Dijkstra's algorithm to find the shortest path from a single source vertex to all other vertices in the given graph.

Algorithm

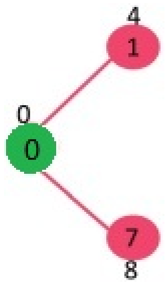
1) Create a set *sptSet* (shortest path tree set) that keeps track of vertices included in shortest path tree, i.e., whose minimum distance from source is calculated and finalized. Initially, this set is empty.
2) Assign a distance value to all vertices in the input graph. Initialize all distance values as INFINITE. Assign distance value as 0 for the source vertex so that it is picked first.
3) While *sptSet* doesn't include all vertices

….**a)** Pick a vertex u which is not there in *sptSet* and has minimum distance value.
….**b)** Include u to *sptSet*.

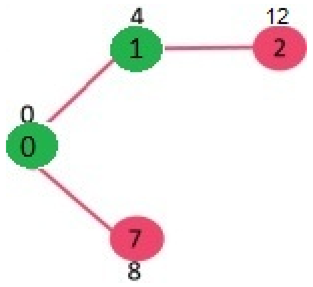
….**c)** Update distance value of all adjacent vertices of u. To update the distance values, iterate through all adjacent vertices. For every adjacent vertex v, if sum of distance value of u (from source) and weight of edge u-v, is less than the distance value of v, then update the distance value of v.



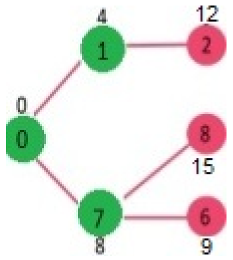
The set *sptSet* is initially empty and distances assigned to vertices are {0, INF, INF, INF, INF, INF, INF, INF} where INF indicates infinite. Now pick the vertex with minimum distance value. The vertex 0 is picked, include it in *sptSet*. So *sptSet* becomes {0}. After including 0 to *sptSet*, update distance values of its adjacent vertices. Adjacent vertices of 0 are 1 and 7. The distance values of 1 and 7 are updated as 4 and 8. Following subgraph shows vertices and their distance values, only the vertices with finite distance values are shown. The vertices included in SPT are shown in green colour.



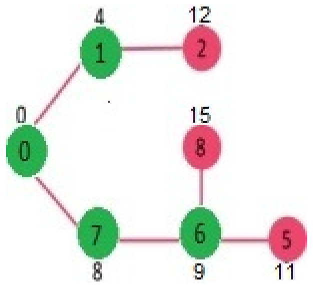
Pick the vertex with minimum distance value and not already included in SPT (not in sptSET). The vertex 1 is picked and added to sptSet. So sptSet now becomes {0, 1}. Update the distance values of adjacent vertices of 1. The distance value of vertex 2 becomes 12.



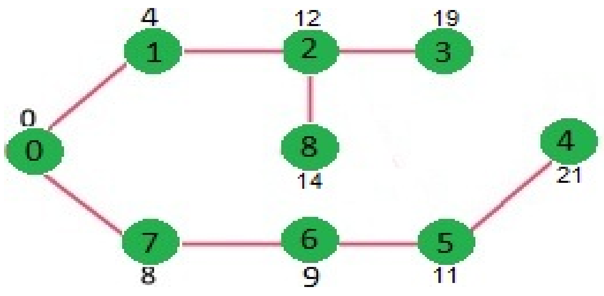
Pick the vertex with minimum distance value and not already included in SPT (not in sptSET). Vertex 7 is picked. So sptSet now becomes {0, 1, 7}. Update the distance values of adjacent vertices of 7. The distance value of vertex 6 and 8 becomes finite (15 and 9 respectively).



Pick the vertex with minimum distance value and not already included in SPT (not in sptSET). Vertex 6 is picked. So sptSet now becomes {0, 1, 7, 6}. Update the distance values of adjacent vertices of 6. The distance value of vertex 5 and 8 are updated.



We repeat the above steps until *sptSet* doesn't include all vertices of given graph. Finally, we get the following Shortest Path Tree (SPT).



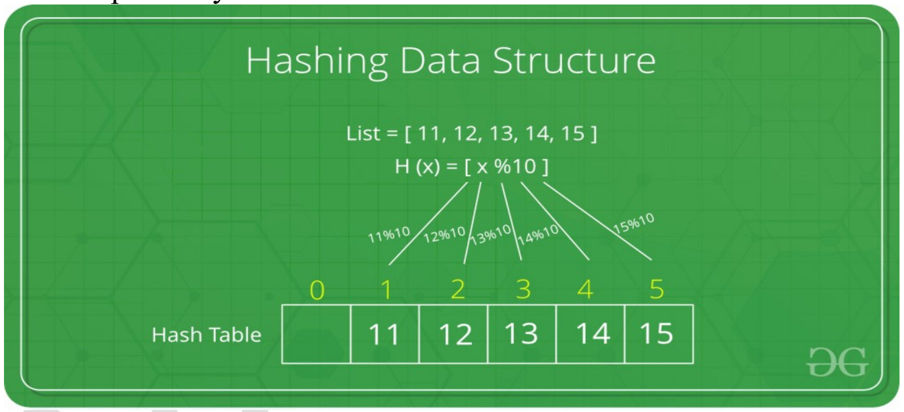
Hashing

Hash Table is a data structure which stores data in an associative manner. In a hash table, data is stored in an array format, where each data value has its own unique index value. Access of data becomes very fast if we know the index of the desired data.

Thus, it becomes a data structure in which insertion and search operations are very fast irrespective of the size of the data. Hash Table uses an array as a storage medium and uses hash technique to generate an index where an element is to be inserted or is to be located from.

Hashing is an important Data Structure which is designed to use a special function called the Hash function which is used to map a given value with a particular key for faster access of elements. The efficiency of mapping depends of the efficiency of the hash function used.

Let a hash function H(x) maps the value at the index **x%10** in an Array. For example if the list of values is [11,12,13,14,15] it will be stored at positions {1,2,3,4,5} in the array or Hash table respectively.



UNIT: IV

What is an Algorithm?

An algorithm is a step by step method of solving a problem. It is commonly used for data processing, calculation and other related computer and mathematical operations.

An algorithm is a detailed series of instructions for carrying out an operation or solving a problem

ALGORITHM SPECIFICATION

An algorithm is a finite set of instructions that, if followed, accomplishes a particular task.
All algorithms must satisfy the following criteria: Input.
An algorithm has zero or more inputs, taken from a specified set of objects.
Output. An algorithm has one or more outputs, which have a specified relation to the inputs.
Definiteness. Each step must be precisely defined; Each instruction is clear and unambiguous.
Finiteness. The algorithm must always terminate after a finite number of steps.
Effectiveness. All operations to be performed must be sufficiently basic that they can be done exactly and in finite length.

We can describe an algorithm in many ways.

1. Natural language: use a natural language like English
2.Flow charts: Graphic representations called flowcharts , only if the algorithm is small and simple.
3.Pseudo-code: also avoids most issues of ambiguity; no particularity on syntax programming language

Example 1: Algorithm for calculate factorial value of a number:

```
 input a number n
 set variable final as 1
final <= final * n
decrease n
check if n is equal to 0
if n is equal to zero, goto step 8 (break out of loop)
 else goto step 3
print the result final
```

Algorithm Analysis

Efficiency of an algorithm can be analyzed at two different stages, before implementation and after implementation, as

A priori analysis − This is theoretical analysis of an algorithm. Efficiency of algorithm is measured by assuming that all other factors e.g. processor speed, are constant and have no effect on implementation.

A posterior analysis − This is empirical analysis of an algorithm. The selected algorithm is implemented using programming language. This is then executed on target computer machine. In this analysis, actual statistics like running time and space required are collected.

Algorithm analysis deals with the execution or running time of various operations involved. Running time of an operation can be defined as no. of computer instructions executed per operation.

Algorithm Complexity

Suppose X is an algorithm and n is the size of input data, the time and space used by the Algorithm X are the two main factors which decide the efficiency of X. Time Factor − The time is measured by counting the number of key operations such as comparisons in sorting algorithm Space Factor − The space is measured by counting the maximum memory space required by the algorithm.

The complexity of an algorithm f(n) gives the running time and / or storage space required by the algorithm in terms of n as the size of input data.

1)Space Complexity:

Space complexity of an algorithm represents the amount of memory space required by the algorithm in its life cycle. Space required by an algorithm is equal to the sum of the following two components:

A fixed part that is a space required to store certain data and variables, that are independent of the size of the problem. For example simple variables & constant used, program size etc.

A variable part is a space required by variables, whose size depends on the size of the problem. For example dynamic memory allocation, recursion stack space etc.

Space complexity S(P) of any algorithm P is S(P) = C + SP(I) Where C is the fixed part and S(I) is the variable part of the algorithm which depends on instance characteristic I. Following is a simple example that tries to explain the concept.

```
Algorithm: SUM(A, B)
Step 1 - START
Step 2 - C ← A + B + 10
Step 3 – Stop
```

Here we have three variables A, B and C and one constant. Hence S(P) = 1+3. Now space depends on data types of given variables and constant types and it will be multiplied accordingly.

2)Time Complexity:

Time Complexity of an algorithm represents the amount of time required by the algorithm to run to completion. Time requirements can be defined as a numerical function T(n), where T(n) can be measured as the number of steps, provided each step consumes constant time.

For example, addition of two n-bit integers takes n steps. Consequently, the total computational time is T(n) = c*n, where c is the time taken for addition of two bits. Here, we observe that T(n) grows linearly as input size increases.

Asymptotic Analysis:

Asymptotic analysis of an algorithm, refers to defining the mathematical framing of its run-time performance. Using asymptotic analysis, we can conclude the best case, average case and worst case scenario of an algorithm.

Asymptotic analysis are input bound i.e., if there's no input to the algorithm it is concluded to work in a constant time. Other than the "input" all other factors are considered constant.

Asymptotic analysis refers to computing the running time of any operation in mathematical units of computation. For example, running time of one operation is computed as f(n) and may be for another operation it is computed as g(n2). Which means first operation running time will increase linearly with the increase in n and running time of second operation will increase exponentially when n increases.

Usually, time required by an algorithm falls under three types –

Best Case − Minimum time required for program execution.
Average Case − Average time required for program execution.
Worst Case − Maximum time required for program execution.

Asymptotic Notations:

Following are commonly used asymptotic notations used in calculating running time complexity of an algorithm.

O(big O) Notation
Ω (Omega)Notation
θ (Theta)Notation

Big Oh Notation, O:
The O(n) is the way to express the upper bound of an algorithm's running time. It measures the worst case time complexity or longest amount of time an algorithm can possibly take to complete.

Omega Notation, Ω:
The Ω(n) is the way to express the lower bound of an algorithm's running time. It measures the best case time complexity or best amount of time an algorithm can possibly take to complete.

Theta Notation, θ:
The θ(n) is the way to express both the lower bound and upper bound of an algorithm's running time.

Common Asymptotic Notations :

Constant	− O(1)
Logarithmic	− O(log n)
Linear	− O(n)

n log n − O(n log n)
Quadratic − O(^{n2})
Cubic − O(^{n3})
Polynomial − nO(1)
Exponential − O(2 ^{n})

Primality Testing

A natural number N is said to be a prime number if it can be divided only by 1 and itself. Primality Testing is done to check if a number is a prime or not. The topic explains different algorithms available for primality testing.

Basic Method:

This is an approach that goes in a way to convert definition of prime numbers to code. It checks if any of the number less than a given number(N) divides the number or not. But on observing the factors of any number, this method can be limited to check only till N. This is because, product of any two numbers greater than N can never be equal to N. A C++ function for basic method is shown below.

```
int PrimeTest(int N)
{
   for (int i = 2; i*i <= N; ++i)
   {
      if(N%i == 0)
      {
         return 0;
      }
   }
   return 1;
}
```

The function returns 1 if N is a prime number and 0 for a composite number. This function runs with a complexity of O(n). That implies, this method can at most be used for numbers of range 1015 to 1016 to determine if it's a prime or not in reasonable amount of time.

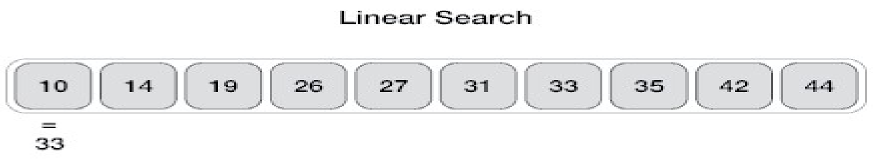
Two examples of primality testing

1) Fermat Primality Testing
2) Miller-Rabin Primality Testing

Searching:

Linear search

Linear search is a very simple search algorithm. In this type of search, a sequential search is made over all items one by one. Every item is checked and if a match is found then that particular item is returned, otherwise the search continues till the end of the data collection.



Algorithm

```
Linear Search ( Array A, Value x)

Step 1: Set i to 1
Step 2: if i > n then go to step 7
Step 3: if A[i] = x then go to step 6
Step 4: Set i to i + 1
Step 5: Go to Step 2
Step 6: Print Element x Found at index i and go to step 8
Step 7: Print element not found
Step 8: Exit
```

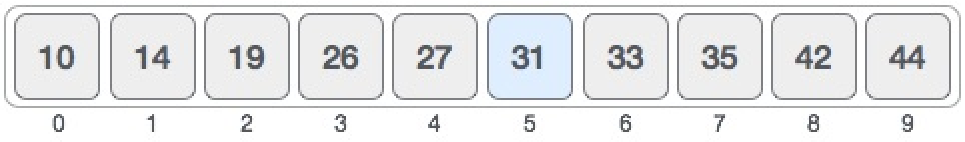
Binary search

Binary search is a fast search algorithm with run-time complexity of O(log n). This search algorithm works on the principle of divide and conquer. For this algorithm to work properly, the data collection should be in the sorted form.

Binary search looks for a particular item by comparing the middle most item of the collection. If a match occurs, then the index of item is returned. If the middle item is greater than the item, then the item is searched in the sub-array to the left of the middle item. Otherwise, the item is searched for in the sub-array to the right of the middle item. This process continues on the sub-array as well until the size of the sub array reduces to zero.

How Binary Search Works?

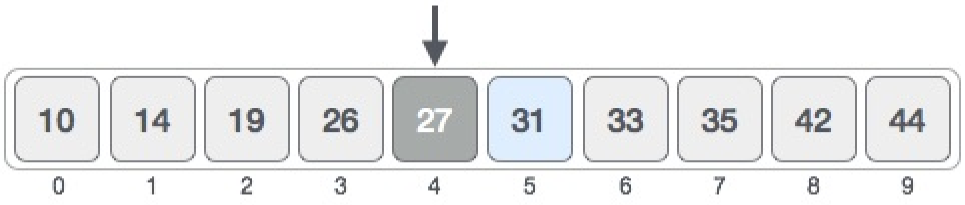
For a binary search to work, it is mandatory for the target array to be sorted. We shall learn the process of binary search with a pictorial example. The following is our sorted array and let us assume that we need to search the location of value 31 using binary search.



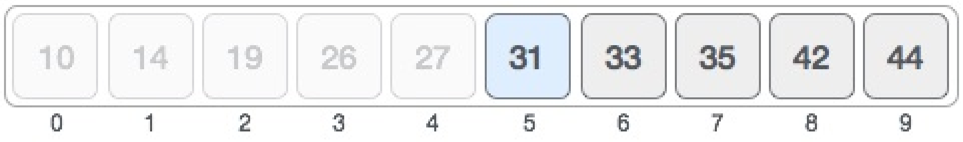
First, we shall determine half of the array by using this formula −

```
mid = low + (high - low) / 2
```

Here it is, 0 + (9 - 0) / 2 = 4 (integer value of 4.5). So, 4 is the mid of the array.



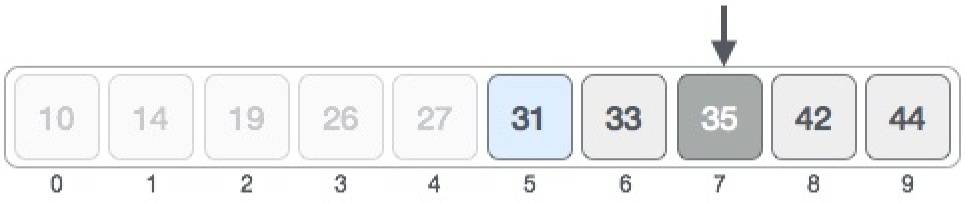
Now we compare the value stored at location 4, with the value being searched, i.e. 31. We find that the value at location 4 is 27, which is not a match. As the value is greater than 27 and we have a sorted array, so we also know that the target value must be in the upper portion of the array.



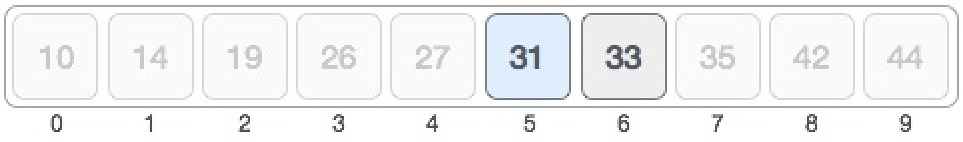
We change our low to mid + 1 and find the new mid value again.

```
low = mid + 1
mid = low + (high - low) / 2
```

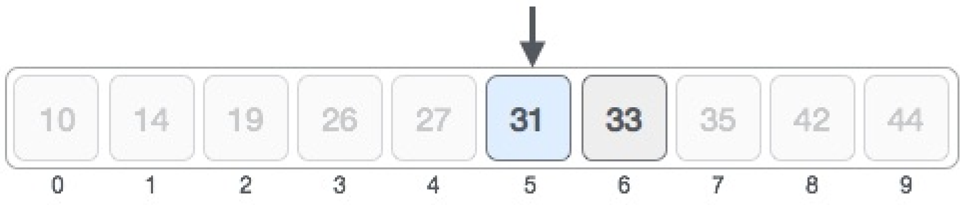
Our new mid is 7 now. We compare the value stored at location 7 with our target value 31.



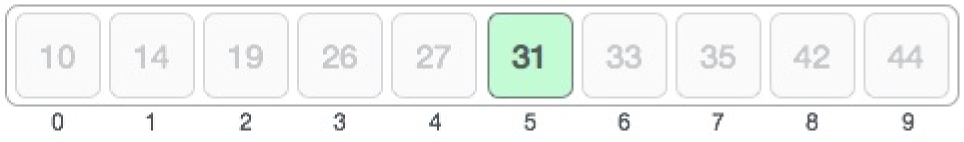
The value stored at location 7 is not a match, rather it is more than what we are looking for. So, the value must be in the lower part from this location.



Hence, we calculate the mid again. This time it is 5.



We compare the value stored at location 5 with our target value. We find that it is a match.



We conclude that the target value 31 is stored at location 5.

Binary search halves the searchable items and thus reduces the count of comparisons to be made to very less numbers.

Sorting :

Sorting refers to arranging data in a particular format. Sorting algorithm specifies the way to arrange data in a particular order. Most common orders are in numerical or lexicographical order.

The importance of sorting lies in the fact that data searching can be optimized to a very high level, if data is stored in a sorted manner. Sorting is also used to represent data in more readable formats. Following are some of the examples of sorting in real-life scenarios −

- **Telephone Directory** − The telephone directory stores the telephone numbers of people sorted by their names, so that the names can be searched easily.
- **Dictionary** − The dictionary stores words in an alphabetical order so that searching of any word becomes easy.

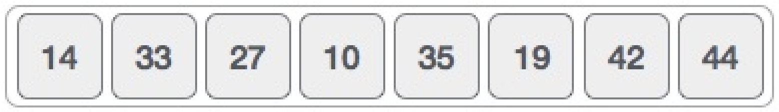
Merge sort

Merge sort is a sorting technique based on divide and conquer technique. With worst-case time complexity being O(n log n), it is one of the most respected algorithms.

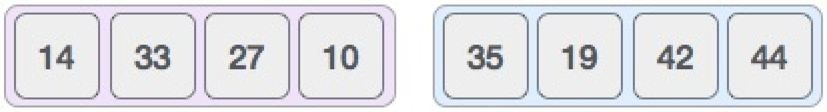
Merge sort first divides the array into equal halves and then combines them in a sorted manner.

How Merge Sort Works?

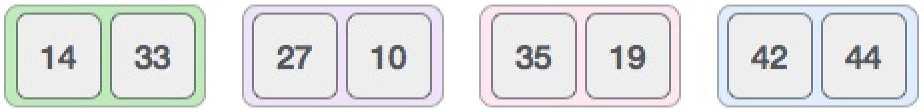
To understand merge sort, we take an unsorted array as the following −



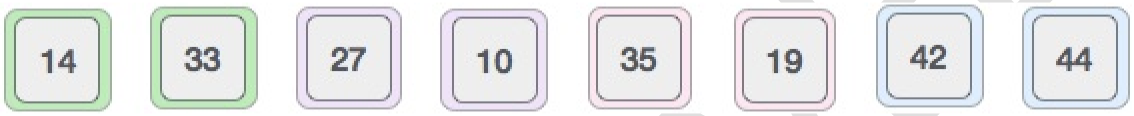
We know that merge sort first divides the whole array iteratively into equal halves unless the atomic values are achieved. We see here that an array of 8 items is divided into two arrays of size 4.



This does not change the sequence of appearance of items in the original. Now we divide these two arrays into halves.

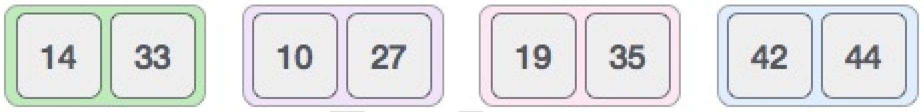


We further divide these arrays and we achieve atomic value which can no more be divided.

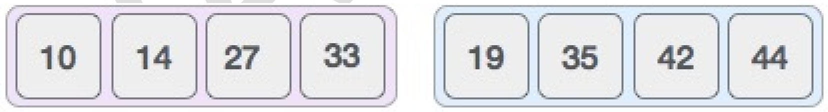


Now, we combine them in exactly the same manner as they were broken down. Please note the color codes given to these lists.

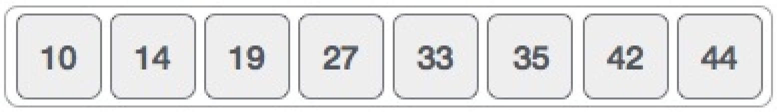
We first compare the element for each list and then combine them into another list in a sorted manner. We see that 14 and 33 are in sorted positions. We compare 27 and 10 and in the target list of 2 values we put 10 first, followed by 27. We change the order of 19 and 35 whereas 42 and 44 are placed sequentially.



In the next iteration of the combining phase, we compare lists of two data values, and merge them into a list of found data values placing all in a sorted order.



After the final merging, the list should look like this −



Now we should learn some programming aspects of merge sorting.

Algorithm

Merge sort keeps on dividing the list into equal halves until it can no more be divided. By definition, if it is only one element in the list, it is sorted. Then, merge sort combines the smaller sorted lists keeping the new list sorted too.

```
Step 1 − if it is only one element in the list it is already sorted, return.
Step 2 − divide the list recursively into two halves until it can no more be divided.
Step 3 − merge the smaller lists into new list in sorted order.
```

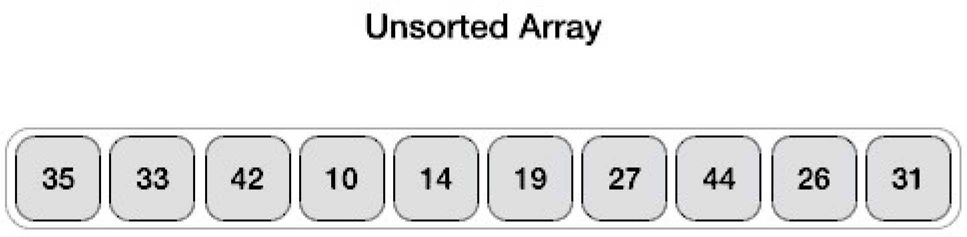
Quick Sort

Quick sort is a highly efficient sorting algorithm and is based on partitioning of array of data into smaller arrays. A large array is partitioned into two arrays one of which holds values smaller than the specified value, say pivot, based on which the partition is made and another array holds values greater than the pivot value.

Quick sort partitions an array and then calls itself recursively twice to sort the two resulting subarrays. This algorithm is quite efficient for large-sized data sets as its average and worst case complexity are of O(n^2), where **n** is the number of items.

Partition in Quick Sort

Following animated representation explains how to find the pivot value in an array.



The pivot value divides the list into two parts. And recursively, we find the pivot for each sub-lists until all lists contains only one element.

Quick Sort Pivot Algorithm

Based on our understanding of partitioning in quick sort, we will now try to write an algorithm for it, which is as follows.

```
Step 1 − Choose the highest index value has pivot
Step 2 − Take two variables to point left and right of the list excluding pivot
Step 3 − left points to the low index
Step 4 − right points to the high
Step 5 − while value at left is less than pivot move right
Step 6 − while value at right is greater than pivot move left
Step 7 − if both step 5 and step 6 does not match swap left and right
Step 8 − if left ≥ right, the point where they met is new pivot
```

Quick Sort Algorithm

Using pivot algorithm recursively, we end up with smaller possible partitions. Each partition is then processed for quick sort. We define recursive algorithm for quicksort as follows −

```
Step 1 − Make the right-most index value pivot
Step 2 − partition the array using pivot value
Step 3 − quicksort left partition recursively
Step 4 − quicksort right partition recursively
```

External Sorting

External sorting is a term for a class of sorting algorithms that can handle massive amounts of data. External sorting is required when the data being sorted do not fit into the main memory of a computing device (usually RAM) and instead they must reside in the slower external memory (usually a hard drive). External sorting typically uses a hybrid sort-merge strategy. In the sorting phase, chunks of data small enough to fit in main memory are read, sorted, and written out to a temporary file. In the merge phase, the sorted sub-files are combined into a single larger file.

One example of external sorting is the external merge sort algorithm, which sorts chunks that each fit in RAM, then merges the sorted chunks together. We first divide the file into **runs** such that the size of a run is small enough to fit into main memory. Then sort each run in main memory using merge sort sorting algorithm. Finally merge the resulting runs together into successively bigger runs, until the file is sorted.

Unit V

Greedy Method:

Greedy algorithms build a solution part by part, choosing the next part in such a way, that it gives an immediate benefit. This approach never reconsiders the choices taken previously. This approach is mainly used to solve optimization problems.

Knapsack Problem

Given a set of items, each with a weight and a value, determine a subset of items to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.

The knapsack problem is in combinatorial optimization problem. It appears as a subproblem in many, more complex mathematical models of real-world problems. One general approach to difficult problems is to identify the most restrictive constraint, ignore the others, solve a knapsack problem, and somehow adjust the solution to satisfy the ignored constraints.

Problem Scenario

A thief is robbing a store and can carry a maximal weight of ***W***into his knapsack. There are n items available in the store and weight of i^{th} item is w_i and its profit is p_i. What items should the thief take?

In this context, the items should be selected in such a way that the thief will carry those items for which he will gain maximum profit. Hence, the objective of the thief is to maximize the profit.

Example

Let us consider that the capacity of the knapsack ***W* = 60** and the list of provided items are shown in the following table

Item	A	B	C	D
Profit	280	100	120	120
Weight	40	10	20	24
Ratio (piwi)(piwi)	7	10	6	5

As the provided items are not sorted based on pi/wi. After sorting, the items are as shown in the following table.

Item	B	A	C	D
Profit	100	280	120	120
Weight	10	40	20	24
Ratio ($\frac{p_i}{w_i}$)(piwi)	10	7	6	5

Solution

After sorting all the items according to pi/wi. First all of ***B*** is chosen as weight of ***B*** is less than the capacity of the knapsack. Next, item ***A*** is chosen, as the available capacity of the knapsack is greater than the weight of ***A***. Now, ***C*** is chosen as the next item. However, the whole item cannot be chosen as the remaining capacity of the knapsack is less than the weight of ***C***.
Hence, fraction of ***C*** (i.e. (60 − 50)/20) is chosen.

Now, the capacity of the Knapsack is equal to the selected items. Hence, no more item can be selected.

The total weight of the selected items is **10 + 40 + 20 * (10/20) = 60**

And the total profit is **100 + 280 + 120 * (10/20) = 380 + 60 = 440**

This is the optimal solution. We cannot gain more profit selecting any different combination of items.

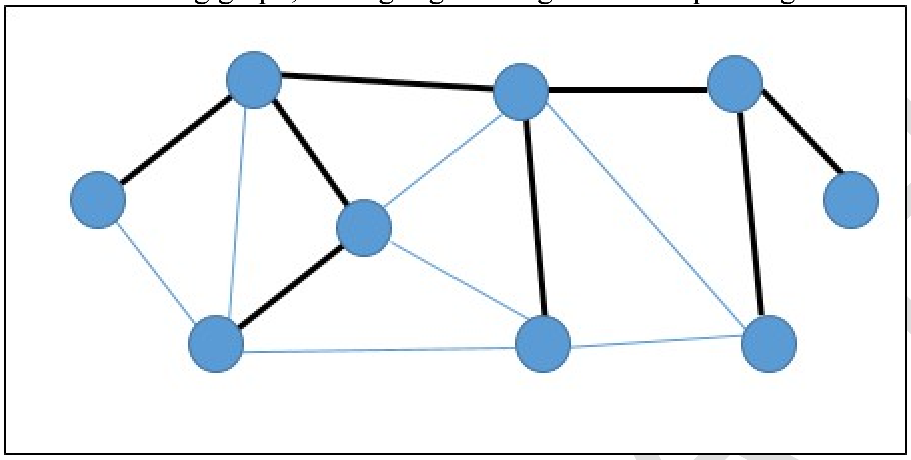
Algorithm: Greedy-Fractional-Knapsack (w[1..n], p[1..n], W)

```
for i = 1 to n
  do x[i] = 0
weight = 0
for i = 1 to n
  if weight + w[i] ≤ W then
    x[i] = 1
    weight = weight + w[i]
  else
    x[i] = (W - weight) / w[i]
    weight = W
    break
return x
```

A **spanning tree** is a subset of an undirected Graph that has all the vertices connected by minimum number of edges.

If all the vertices are connected in a graph, then there exists at least one spanning tree. In a graph, there may exist more than one spanning tree.

Properties
A spanning tree does not have any cycle.
Any vertex can be reached from any other vertex.
Example
In the following graph, the highlighted edges form a spanning tree.

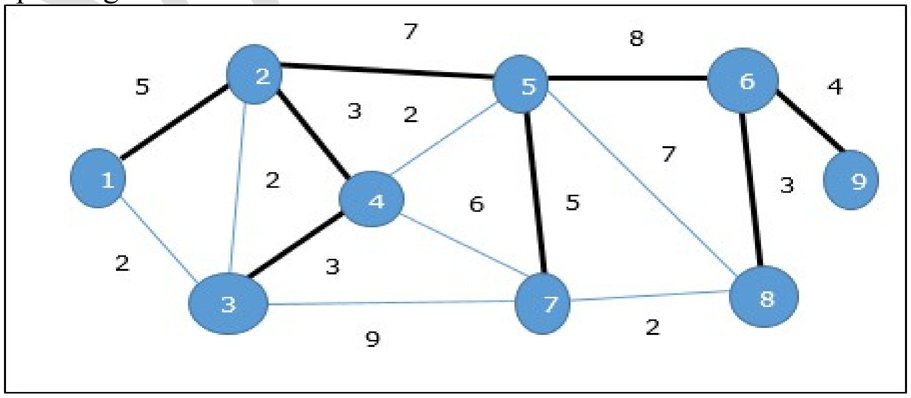


Minimum Spanning Tree

A **Minimum Spanning Tree (MST)** is a subset of edges of a connected weighted undirected graph that connects all the vertices together with the minimum possible total edge weight. To derive an MST, Prim's algorithm or Kruskal's algorithm can be used. Hence, we will discuss Prim's algorithm in this chapter.

As we have discussed, one graph may have more than one spanning tree. If there are **n** number of vertices, the spanning tree should have ***n* - 1** number of edges. In this context, if each edge of the graph is associated with a weight and there exists more than one spanning tree, we need to find the minimum spanning tree of the graph.

Moreover, if there exist any duplicate weighted edges, the graph may have multiple minimum spanning tree.



In the above graph, we have shown a spanning tree though it's not the minimum spanning tree. The cost of this spanning tree is (5 + 7 + 3 + 3 + 5 + 8 + 3 + 4) = 38.

We will use Prim's algorithm to find the minimum spanning tree.

Prim's Algorithm

Prim's algorithm is a greedy approach to find the minimum spanning tree. In this algorithm, to form a MST we can start from an arbitrary vertex.

```
Algorithm: MST-Prim's (G, w, r)
for each u ϵ G.V
  u.key = ∞
  u.∏ = NIL
r.key = 0
Q = G.V
while Q ≠ Φ
  u = Extract-Min (Q)
  for each v ϵ G.adj[u]
    if each v ϵ Q and w(u, v) < v.key
      v.∏ = u
      v.key = w(u, v)
```

The function Extract-Min returns the vertex with minimum edge cost. This function works on min-heap.

Example

Using Prim's algorithm, we can start from any vertex, let us start from vertex **1**.

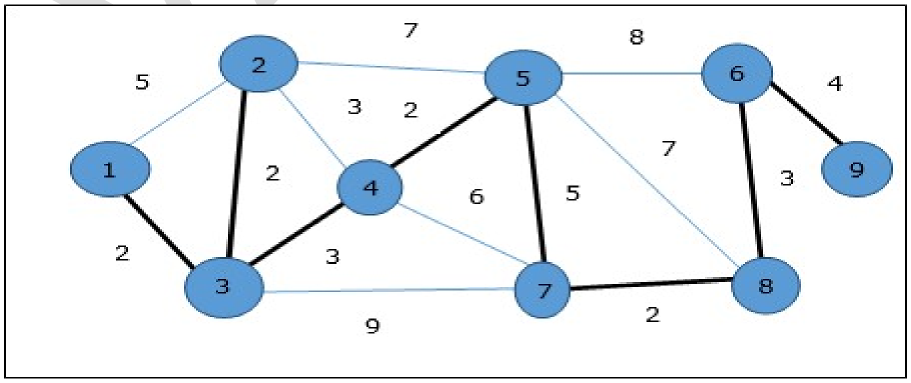
Vertex **3** is connected to vertex **1** with minimum edge cost, hence edge **(1, 2)** is added to the spanning tree.

Next, edge **(2, 3)** is considered as this is the minimum among edges **{(1, 2), (2, 3), (3, 4), (3, 7)}**.

In the next step, we get edge **(3, 4)** and **(2, 4)** with minimum cost. Edge **(3, 4)** is selected at random.

In a similar way, edges **(4, 5), (5, 7), (7, 8), (6, 8)** and **(6, 9)**are selected. As all the vertices are visited, now the algorithm stops.

The cost of the spanning tree is (2 + 2 + 3 + 2 + 5 + 2 + 3 + 4) = 23. There is no more spanning tree in this graph with cost less than **23**.



Travelling salesman problem

Problem Statement

A traveler needs to visit all the cities from a list, where distances between all the cities are known and each city should be visited just once. What is the shortest possible route that he visits each city exactly once and returns to the origin city?

Solution

Travelling salesman problem is the most notorious computational problem. We can use brute-force approach to evaluate every possible tour and select the best one. For **n** number of vertices in a graph, there are **(*n* - 1)!** number of possibilities.

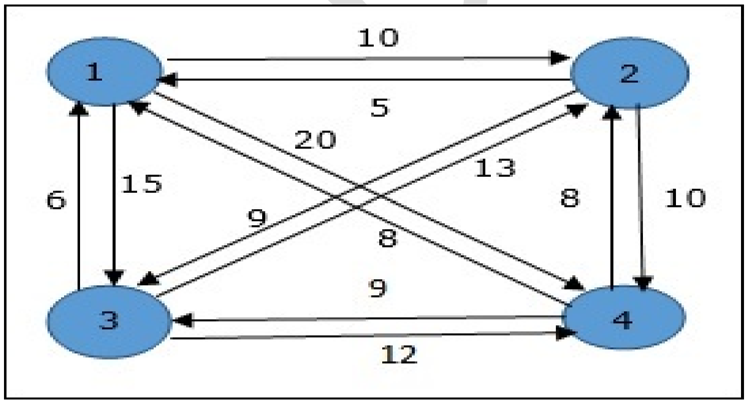
Instead of brute-force using dynamic programming approach, the solution can be obtained in lesser time, though there is no polynomial time algorithm.

Let us consider a graph ***G* = *(V, E)***, where ***V*** is a set of cities and ***E*** is a set of weighted edges. An edge ***e(u, v)*** represents that vertices ***u*** and ***v*** are connected. Distance between vertex ***u*** and ***v*** is ***d(u, v)***, which should be non-negative.

Suppose we have started at city ***1*** and after visiting some cities now we are in city ***j***. Hence, this is a partial tour. We certainly need to know ***j***, since this will determine which cities are most convenient to visit next. We also need to know all the cities visited so far, so that we don't repeat any of them. Hence, this is an appropriate sub-problem.

Example

In the following example, we will illustrate the steps to solve the travelling salesman problem.



From the above graph, the following table is prepared.

	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

```
Algorithm: Traveling-Salesman-Problem
C ({1}, 1) = 0
for s = 2 to n do
   for all subsets S Є {1, 2, 3, … , n} of size s and containing 1
      C (S, 1) = ∞
   for all j Є S and j ≠ 1
      C (S, j) = min {C (S – {j}, i) + d(i, j) for i Є S and i ≠ j}
Return minj C ({1, 2, 3, …, n}, j) + d(j, i)
```

Backtracking Algorithms

Backtracking is an algorithmic-technique for solving problems recursively by trying to build a solution incrementally, one piece at a time, removing those solutions that fail to satisfy the constraints of the problem at any point of time (by time, here, is referred to the time elapsed till reaching any level of the search tree).

8 queen problem

The eight queens problem is the problem of placing eight queens on an 8×8 chessboard such that none of them attack one another (no two are in the same row, column, or diagonal). More generally, the n queens problem places n queens on an n×n chessboard.

Algorithm

isValid(board, row, col)
Input: The chess board, row and the column of the board.
Output: True when placing a queen in row and place position is a valid or not.

```
Begin
   if there is a queen at the left of current col, then
      return false
   if there is a queen at the left upper diagonal, then
      return false
   if there is a queen at the left lower diagonal, then
      return false;
   return true //otherwise it is valid place
End
```

NQueen(board, col)

Input: The chess board, the col where the queen is trying to be placed.

Output: The position matrix where queens are placed.

```
Begin
   if all columns are filled, then
      return true
   for each row of the board, do
      if isValid(board, i, col), then
         set queen at place (i, col) in the board
         if solveNQueen(board, col+1) = true, then
            return true
         otherwise remove queen from place (i, col) from board.
   done
   return false
End
```